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Diversification Strategies in African Frontier Markets: An Empirical Back-Testing Approach

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ABSTRACT

In accordance with the result of the back-test in different diversified portfolios in the African frontier market, African frontier markets provide both potential benefits and challenges, which was reasoned by their different degrees of disintegration and volatility from the global capital market. This study seeks to assess the diversification advantages objectively of African frontier markets using a back-testing methodology. The study employed portfolio optimization strategies to evaluate risk-adjusted returns and inter-market correlations, and it is possible to utilize daily log-returns from ten frontier markets spanning 2015 to 2024. The results indicated that diversity in African frontier markets offers substantial risk mitigation; nevertheless, excessive diversification might impact profits negatively. Aiming to optimize the investment benefits, the decision of investment had a demand to concentrate on balancing the integration degrees in portfolios.

KEYWORDS

Frontier markets; Africa; Diversification strategy; Portfolio optimization; Risk-return analysis

1. INTRODUCTION

The observations in this study could be included as the Africa frontier markets; these markets are characterized by rapid economic growth, expanding financial infrastructure, and low integration with developed economies, which together provide potential diversification benefits (Dania and Maysami, 2017; Evans and van Vuuren, 2019) [1, 2]. Investors looking to control their risk and improve the efficiency of their portfolios prefer these markets because they lack strong links to global capital markets.

Although these market observations have potential benefits, they also have downsides. Previous studies indicated that these markets often encounter issues including political instability, inadequate liquidity, regulatory inefficiencies, and high transaction costs; these factors undermined their reliability as investment platforms potentially (Marshall et al., 2013; Ferreira et al., 2021) [3, 4]. Additionally, a lot of existing research has remained at the theoretical level, focusing on the conceptual and theoretical potential of diversity but lacking empirical evidence. There is still no systematic back-testing to confirm the accuracy of these theoretical benefits throughout multiple economic cycles (Fifield et al., 2002) [5].

Aiming to obtain the maximum-benefit portfolio strategy in African frontier markets, this study used a back-testing methodology. It examined the impact of mixing assets across different markets on overall risk exposure, the effects of portfolio correlations over time, and the potential role of these markets in enhancing global portfolios according to Modern Portfolio Theory.

The significance of this study could be described as it provided extensive empirical evidence concerning the performance of African frontier market portfolios within diversified strategies. The research utilized portfolio optimization algorithms and a quantitative risk-return analysis to elucidate the advantages and drawbacks of investing in Africa's frontier markets.

2. DATA AND METHODOLOGY

2.1. Data Source

The 10 frontier markets included in the sample were Botswana (BOT), Egypt (EGP), Ghana (GHA), Senegal (SEN), Morocco (MOR), Namibia (NAM), Mauritius (MAU), Kenya (KEN), Uganda (UGA), and Zambia (ZAM). The dataset consists of 2,610 observations for each market index, corresponding to 10 years of trading days from 2015 to 2024. Aiming to research the sustainability and risk-resistance capacity of the diversified investment strategy, this sample covers a range of economic environments and events that caused high market volatility (for example, commodity price fluctuations, international geographic political changes in Ukraine, and the COVID-19 pandemic from 2020 to 2022). A number of descriptive statistics were calculated for each market, including the mean, standard error, median, standard deviation, kurtosis, skewness, minimum return, and maximum return.

2.2. Processing Method

In order to research diversification benefits from investing in frontier stock markets, its daily stock index prices for ten frontier markets were obtained for the 10-year period 2015–2024. Daily returns were computed according to the formula:

The log-returns for each national index could be calculated by the following formula:

$$R_{it} = \text{Ln} \left[(P_{it}/P_{it-1}) (X_{t-1}/X_t) \right]$$

Where:

Rit represents the return on share i in day t

Pit represents the price level of the share in day t

Xt represents the exchange rate at time t

Ln represents using the natural logarithm for calculation

This approach of log-return considers the differences in price volatility and currency fluctuations, and provides a composite measurement in returns of USD.

By converting all prices into a common currency, it eliminates exchange rate discrepancies and enables direct comparisons of log returns across different markets. In addition, converting to USD allows for a more standardized comparison by mitigating the impact of varying inflation rates across countries, thereby providing a clearer picture of real returns.

By using Minitab, the expected portfolio return is calculated by multiplying the mean returns by the portfolio weights, while portfolio risk is determined using the ex-post covariance matrix. Additionally, the programmed aims to optimize the MRPUR by selecting the best market combinations for each period. The analysis in this study is solely ex-post, assessing the past performance of different portfolio strategies. The optimal portfolios were determined based on historical data rather than forecast performance, results from running the program are presented in Figure 1. The results confirm that diversification within frontier markets has yielded varying risk-adjusted returns over different periods. Specifically, MOR + NAM + KEN ranks as the best-performing portfolio in Period 1, while the single market investment in CEN was the best portfolio in Period 2; it reflected shifting market conditions.

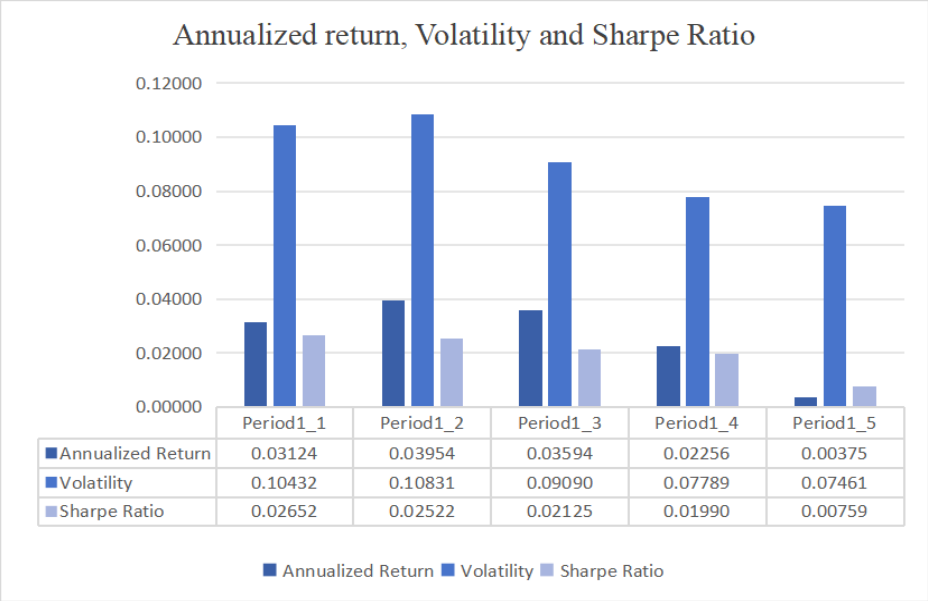


Figure 1. Sum results in different periods

Source: Author's work.

2.3. Key Assumptions

2.3.1. Frontier market efficiency assumption

Generally, frontier markets are not considered to be even weak form efficient. Empirical studies have demonstrated that these markets often exhibit inefficiencies due to factors such as limited liquidity and regulatory constraints. These inefficient factors may be exploited as opportunities for investors, which suggests that frontier markets remain segmented from global capital flows, and allows for periods of mispricing and return anomalies. (Phanrattinon et al., 2020) [6].

2.3.2. Return distribution assumption

Stock returns in frontier markets do not follow a normal distribution. Instead, they tend to exhibit negative skewness and high kurtosis, it illustrates the presence of asymmetric return structures and frequent extreme price fluctuations. (Naumoski et al., 2017) [7].

2.3.3. Currency conversion assumption

All market prices were converted to a common currency (USD) to ensure comparability across different samples. Through standardizing returns in a single currency, distortions caused by exchange rate fluctuations and inflation differentials are eliminated and an accurate assessment of market performance is possible.

2.3.4. Market segmentation and diversification assumption

It is assumed that frontier markets are not integrated with global financial markets. Following MPT, the segmentation assumption creates opportunities for portfolio diversification as returns from these markets move independently of major global indices.

3. RESULTS

In order to evaluate the long-term dynamics of African frontier markets, it is essential to focus on their return performance in terms of their statistic indicators.

3.1. Market Performance for Mean And Median

Mean and median returns (Figure 2 and Figure 3) provide an overview of the long-term market performance of each sample market. The mean return, which measures the average daily price change over the period, the graph indicated that most frontier markets experienced non-profitable or slightly negative average returns. This result has alignment with empirical findings on emerging and frontier markets, where long-term returns have a high volatility and are dependent on economic cycles. (Guney et al., 2017) [8].

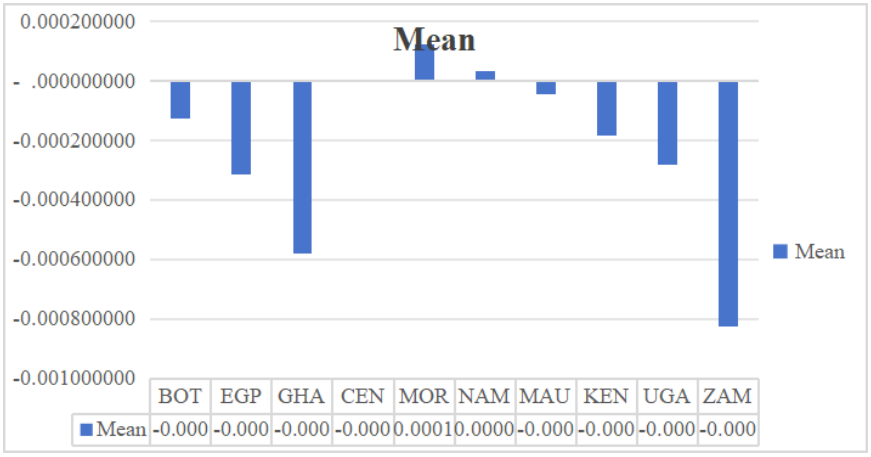


Figure 2. The bar chart of observations means

Source: Author's work.

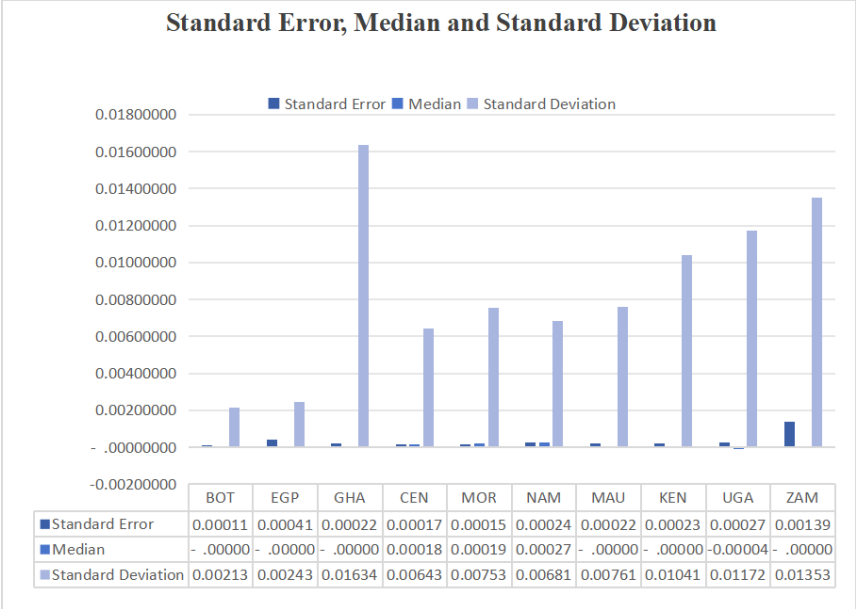


Figure 3. The standard error, median and standard deviation of observations

Source: Author's work.

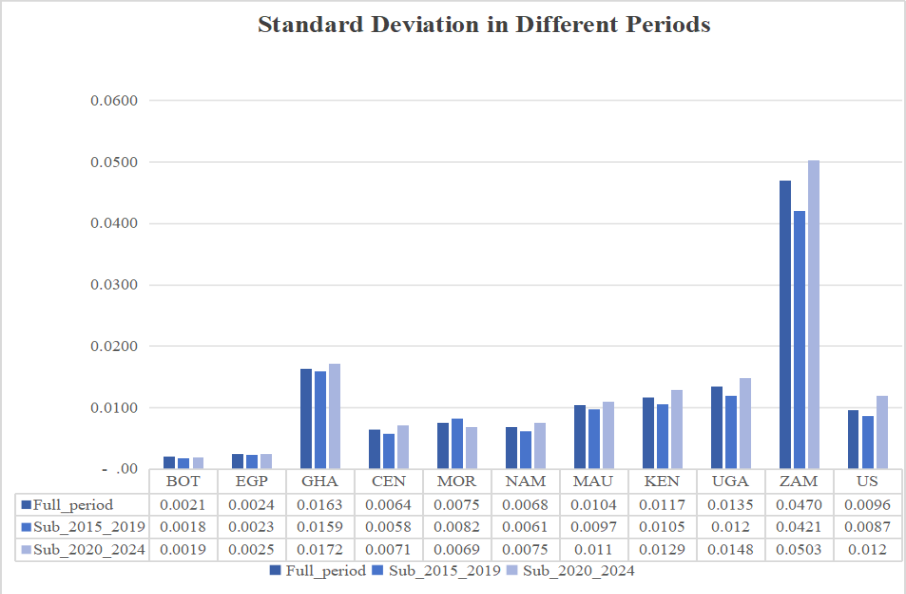
Notably, Morocco (MOR) had the highest mean return (0.00012), while Zambia (ZAM) had the lowest (-0.00082). These figures suggest that Morocco provided slightly better risk-adjusted returns over the period, while Zambia's negative return highlighted its poor market performance and relatively high-risk nature. Concerning the range is small and all mean returns approaching to 0, it illustrated that the overall performance of sample frontier markets exhibited a low long-term return,

suggesting that the investor demand of diversified investment portfolios rather than expecting substantial individual market gains.

The median return represents the midpoint of the return distribution. Most of the sample markets are approaching zero, further confirming that return distributions are not normally distributed. Almost all sample markets show that the median and mean returns diverge significantly, suggesting asymmetry in return distributions, a factor that is further confirmed in the skewness analysis. Morocco (MOR) and Namibia (NAM) have slightly positive medians; it indicates that most of their daily returns were positive, whereas other markets had more frequent negative returns.

3.2. Volatility

Volatility is a key measure of market risk , as it reflects the degree of fluctuation in price changes over time. Standard deviation (Figure 3 and Figure 4) are the metrics used to quantify volatility; a higher standard deviation indicates it would be more uncertain and that there is potential for both gains and losses.



	BOT	EGP	GHA	CEN	MOR	NAM	MAU	KEN	UGA	ZAM	US
Full_period	0.0021	0.0024	0.0163	0.0064	0.0075	0.0068	0.0104	0.0117	0.0135	0.0470	0.0096
Sub_2015_2019	0.0018	0.0023	0.0159	0.0058	0.0082	0.0061	0.0097	0.0105	0.012	0.0421	0.0087
Sub_2020_2024	0.0019	0.0025	0.0172	0.0071	0.0069	0.0075	0.011	0.0129	0.0148	0.0503	0.012

Figure 4. Standard deviation in different periods of observations

Source: Author's work.

Figure 3 and 4 shows that Ghana (GHA) and Zambia (ZAM) exhibited the highest-ranking standard deviations, at 0.01634 and 0.01353, respectively; it illustrates that these markets were affected by the largest daily price swings. These findings indicate that investors in these markets face significant uncertainty, which can stem from macroeconomic instability, geographic political risks, and external shocks such as global commodity price movements. In contrast, Botswana (BOT) and Egypt (EGP) exhibited the lowest standard deviations of 0.00214 and 0.00244, indicating that their markets were relatively stable over the period. This reveals that these economies are more sustainable and stable in their macroeconomic policies and have stronger financial regulations. Namibia (NAM) and Morocco (MOR) fall in between, with moderate volatility levels, making their investment opportunities more attractive for investors seeking a balance between risk and return.

Regarding Figure 4, the time-series analysis of volatility further gives a viewpoint of the trend of marginally increased market volatility risk over time. It is obvious that the period 2020-2024, almost all markets standard deviation increased significantly in this sub-period, suggesting a heightened level of market uncertainty. Specifically, Zambia’s standard deviation rose from 0.0421 (2015–2019) to 0.0503 (2020–2024), a trend that might be attributed to the economic instability, currency devaluations, and geographical political challenges (IMF, 2021) [9]. Similarly, Kenya (KEN) and

Uganda (UGA) experienced an increased volatility, raised from 0.0105 to 0.0129 and from 0.012 to 0.0148, respectively, which was reasoned by increased capital flow volatility and exchange rate instability (Narayan & Rehman, 2018) [10]. These empirical findings are consistent with previous literature indicating that frontier markets are particularly vulnerable to external factor shocks, such as changes in global interest rates and cash flows, which have significant effects in amplifying market risks (Abidi et al., 2019) [11].

However, Morocco (MOR) stands out as an exception where its volatility decreased in the later period, declined from 0.0082 to 0.0069. This suggested an improvement in market stability, empirical finds show that was reasoned by the progress in terms of regulatory reforms, economic diversification and increased integration with international financial markets (Pätäri et al., 2019) [12]. Additionally, the decline in volatility could also indicated a stronger investor confidence in profits and more predictable market behavior, reinforcing Morocco's potential in liquidity as a relatively safe investment choice within the frontier markets samples. This trend aligned with the findings that from previous studies, it revealed that few frontier markets, especially in those who had strong governance regulatory and policy reforms, tend to exhibit lower risk over time compared to their peers (Nguyen, 2023) [13].

From the perspective of an investor, the observed volatility trends have significant implications for portfolio diversification. While higher volatility suggests greater risk, it also provides opportunities for high returns, particularly in markets which had large price swings, it might create opportunities for arbitrage and speculative investment (Fama and French, 1992) [14]. The increase in volatility post-2020 suggests that investors in frontier markets should be prepared for heightened risk exposure, particularly in periods of global economic uncertainty (World Bank, 2021) [15]. However, despite the risen in volatility, the low correlations observed in Table 1 (also suggests that incorporating frontier markets into a diversified portfolio may still reduce overall risk, as these markets tend to move independently of each other and of developed markets.

Table 1. Regression results of observations

BOT		EGP		GHA		CEN		MOR		NAM		MAU		KEN		UGA		ZAM	
Mean	-0.00012	Mean	-0.00032	Mean	-0.00058	Mean	-0.00000	Mean	0.00012	Mean	0.00003	Mean	-0.00004	Mean	-0.00019	Mean	-0.00028	Mean	-0.00082
Standard Error	0.00011	Standard Error	0.00042	Standard Error	0.00022	Standard Error	0.00018	Standard Error	0.00015	Standard Error	0.00025	Standard Error	0.00023	Standard Error	0.00024	Standard Error	0.00027	Standard Error	0.00140
Median	- .00000	Median	- .00000	Median	- .00000	Median	0.00018	Median	0.00020	Median	0.00028	Median	- .00000	Median	- .00000	Median	-0.00004	Median	- .00000
Standard Deviation	0.00214	Standard Deviation	0.00244	Standard Deviation	0.01634	Standard Deviation	0.00643	Standard Deviation	0.00754	Standard Deviation	0.00682	Standard Deviation	0.00761	Standard Deviation	0.01041	Standard Deviation	0.01172	Standard Deviation	0.01353
Kurtosis*	* 5.36944	Kurtosis	* 202.99774	Kurtosis	* 57.55784	Kurtosis	* 2.30881	Kurtosis	* 15.92917	Kurtosis	* 48.58864	Kurtosis	* 35.34577	Kurtosis	* 4.17896	Kurtosis	* 72.39612	Kurtosis	* 842.15035
Skewness*	* - 0.24021	Skewness	* -9.13428	Skewness	* 2.45344	Skewness	* 0.19223	Skewness	* - 1.12171	Skewness	* - 1.81024	Skewness	* - 1.21820	Skewness	* - 0.16973	Skewness	* 0.05375	Skewness	* - 22.73674
Minimum	-0.04578	Minimum	-0.49124	Minimum	-0.11020	Minimum	-0.04558	Minimum	-0.08846	Minimum	-0.22743	Minimum	-0.15299	Minimum	-0.07135	Minimum	-0.19656	Minimum	-2.74405
Maximum	0.04352	Maximum	0.12745	Maximum	0.19767	Maximum	0.04946	Maximum	0.05099	Maximum	0.12416	Maximum	0.13144	Maximum	0.07473	Maximum	0.20172	Maximum	0.19645
Skewness/ Standard Error	- 2132.141313		- 21909.72917		11099.62185		1071.663497		- 7403.446997		- 7296.870131		- 5320.381351		- 721.4548115		197.8623462		- 16241.39778
Kurtosis/ Standard Error	47659.01954		486915.8462		260398.2172		12871.26665		105135.2048		195854.826		154369.5172		17763.48504		266516.9164		601568.1406

Source: Author's work.

Table 2 Correlation sheet for daily returns of the 10 frontier markets. Each cell shows the pairwise correlation coefficient between the market in the corresponding row and column. With the exception of a few country pairs that had high correlation (e.g., BOT–NAM), most correlations in the sample of frontier markets are extremely low (most of them approaching 0); it illustrates the opportunities of risk reduction by diversifying portfolio investment in these sample markets.

Table 2. Correlation results of each observation

	BOT	EGP	GHA	CEN	MOR	NAM	MAU	KEN	UGA	ZAM
BOT	1									
EGP	0.016204673	1								
GHA	-0.01439782	0.021035217	1							
CEN	0.241325563	-0.001240462	0.012736431	1						
MOR	0.203443421	0.120574193	0.02818337	0.239359268	1					
NAM	0.644574983	0.028729447	-0.00646017	0.162860667	0.148412198	1				
MAU	0.070443753	0.051305087	0.015345504	0.065698195	0.1101286	0.05282944	1			
KEN	-0.002268045	0.030074542	0.028623696	0.031129582	0.075198427	-0.000137023	0.092639431	1		
UGA	0.054917178	0.023135092	0.068054815	0.013682658	0.022284667	0.025759659	0.028941298	0.099824752	1	
ZAM	-0.007449779	0.023634083	0.006456968	0.006307847	0.041108935	-0.011899311	0.018442144	-0.010540001	-0.002295937	1

Source: Author's work

In general, Figures 3 and Figures 4 illustrate the substantial differences in terms of volatility across the sample frontier markets. Several markets, such as Botswana (BOT) and Egypt (EGY), exhibited relatively low risk, while others like Zambia (ZAM) and Ghana (GHA) experienced extreme fluctuations in price changes. The overall trend of increasing volatility in the 2020–2024 period underscores the significance of monitoring risk dynamics in these markets, essentially in response to global geographic macroeconomic events. Morocco's (MOR) declining volatility presents a counterexample, suggesting that stability may be achieved through structural reforms and economic diversification. These findings reveal the importance of a balanced diversification and risk management strategy when investing in frontier markets, as their market risk could evolve over time.

3.3. Skewness and Kurtosis

To assess the significance of skewness and kurtosis. the skewness-to-standard-error and kurtosis-to-standard-error ratios were used. If the absolute values exceed two, they are considered to be significant at the 95 per cent confidence level. As the results show in Figure 1, all results indicate high significance at this level, demonstrating that the distribution of the data deviates significantly from normality.

By reason of the unpredictability of investor behaviors and external characteristics like unsystematic risks in the capital market, return distributions are rarely normal in financial markets. It means that stock returns do not follow the symmetric bell-shaped distribution assumed in most classical finance models.

Instead, asymmetry (skewness) and extreme movements (kurtosis) appear frequently; it makes the measurement of market risk more complex. The skewness statistics (Figure 5) show that most frontier markets exhibited negative skewness over the sample period studied; it means they had a higher probability of yielding extreme negative returns. Egypt (EGP) (-9.13) and Zambia (ZAM) (-22.74) had extreme negative skewness values, indicating that these markets frequently showed large downward movements. This situation posed a significant risk for investors, as large draw-downs lead to substantial capital losses.

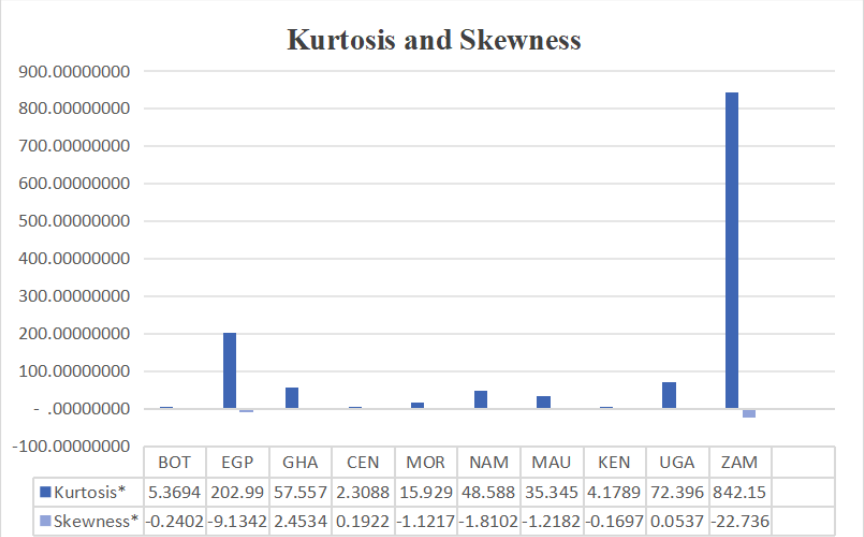


Figure 5. The kurtosis and skewness of observations

Source: Author's work.

However, Ghana (GHA) (2.45) is one of the few markets with positive skewness, suggesting that it had more extreme positive returns than negative in its return distribution. A positively skewed market could be attractive to investors who are looking for a high-upside opportunity, even though it still had a high degree of market risk. The kurtosis values (Figure 5) further highlight the presence of extreme

price movements. According to empirical studies, a kurtosis value above three suggests fat-tailed distributions, meaning that extreme returns (positive and negative) occurs more frequently than in a normal distribution. Zambia (842.15) and Egypt (202.99) exhibited the highest kurtosis values, indicating that their markets frequently experienced extreme price fluctuations. The high kurtosis values reinforce the volatile nature of these markets, and it also reveals their susceptibility to economic shocks, liquidity crises, and abrupt policy changes.

Morocco (MOR) and Kenya (KEN) had relatively moderate kurtosis values (15.92 and 4.18), indicating a lower frequency, although they still had significantly high price volatility. Botswana (BOT) had a kurtosis value of 5.37; it was one of the more stable markets, suggesting a lower tail risk compared with other sample frontier markets.

3.4. Correlation and Diversification Opportunities

Figure 6 illustrates the min-max return range for each market from 2015 to 2024; it shows the degree of daily price movements. Zambia (ZAM) showed the most extreme downward movements; these markets had a minimum return of -2.744%. The downward movement is far more than any other market. Similarly, Egypt (EGP) and Namibia (NAM) showed significant negative spikes, reinforcing their vulnerability to sharp declines. In contrast, Botswana (BOT) and Morocco (MOR) displayed relatively narrow ranges, suggesting greater price stability and lower short-term risk.

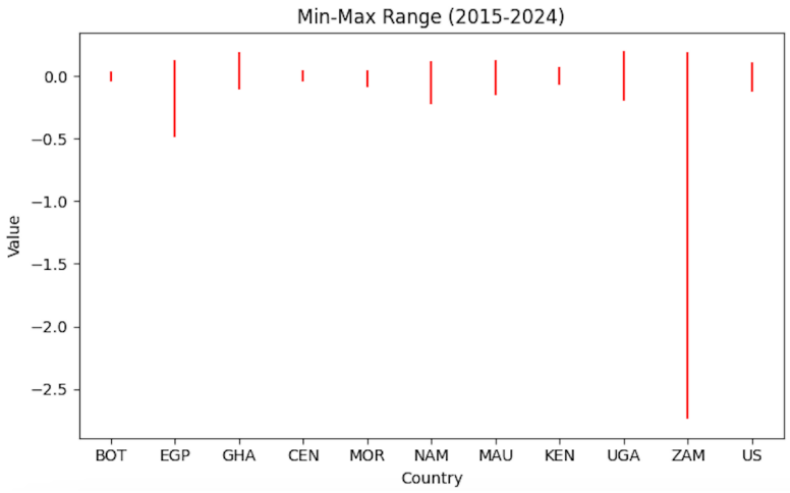


Figure 6. The Min-Max range of observations

Source: Author's work.

These findings indicate that markets with wider return ranges tend to be more volatile; those had alignment with earlier analyses of standard deviation and skewness. Figure 7 revealed the cumulative sum of returns; it indicates long-term market performance. The United States (US) achieved the highest cumulative return, reflecting its sustainable growth in the long term, while Zambia (ZAM) had the worst performance, confirming its long-term decline and weak market performance. Ghana (GHA) and Egypt (EGP) also showed negative cumulative returns, suggesting that their high volatility could not translate into returns. Meanwhile, Morocco (MOR) and Namibia (NAM) maintained slightly positive or near-zero cumulative sums, indicating a relatively stabilized market; they have attracted investors who have a high degree of risk aversion.

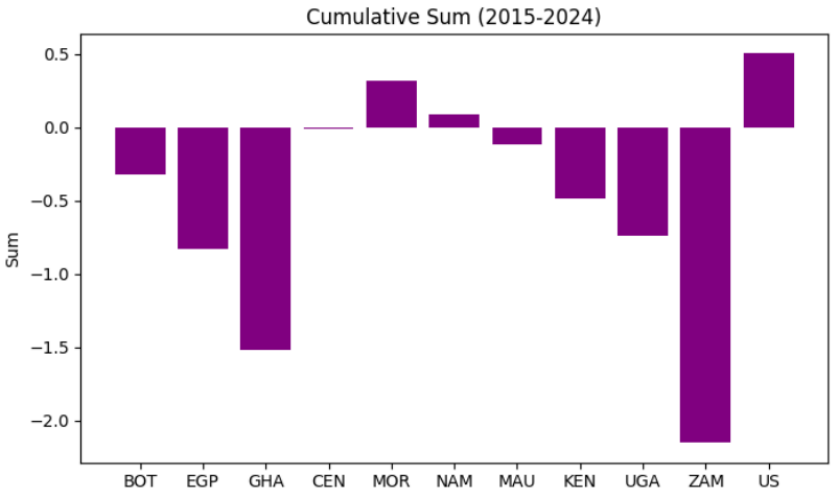


Figure 7. The cumulative sum returns of observations

Source: Author's work.

The relationship between these figures exhibits a key investment principle: a higher volatility does not guarantee higher returns in frontier markets. Some markets, like Zambia and Ghana, experienced extreme fluctuations but failed to generate positive long-term performance. Conversely, the markets that had higher stability, such as Morocco and Namibia, offered more consistent outcomes. Investors should achieve a balance between short-term risk and long-term returns, as excessive volatility may erode profit according to the effect of fear and hope in second-generation behavioral finance theory.

Table 1 illustrates the descriptive statistical results of daily returns for ten African frontier markets over the period 2015–2024. It includes mean, median, standard error, standard deviation, skewness, kurtosis, minimum, and maximum values for each market index. These indicators provide a comprehensive overview of statistical performance and risk characteristics by each market observation.

Figure 8 shows the correlation distribution. The heat-map reveals the correlation matrix of daily log returns of the ten frontier markets visually, the color intensity reflects the strength degree of correlation, with darker shades indicating a stronger positive correlation, while lighter shades and near-zero values indicate weak or no correlations.

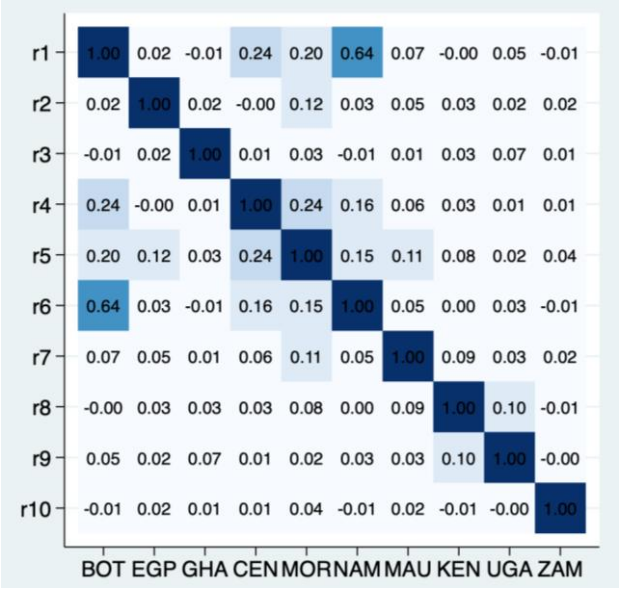


Figure 8. Correlation matrix heat-map

Source: Author's work.

According to Modern Portfolio Theory, holding assets with low or negative correlations helps reduce overall portfolio volatility. In this case, most correlations are close to zero, meaning that combining these frontier markets in a portfolio could significantly lower unsystematic risk.

From Figure 8, which is based on Table 2, it is evident that most frontier markets exhibited low or near-zero correlations; it illustrates that these markets have a weak degree of alignment. This finding provides evidence of diversification benefits from investing across multiple frontier markets, as downturns in one market are less likely to be reflected in others. However, the finding has limitations, with a few exceptions like the Botswana-Namibia (BOT-NAM) pair, which shows a correlation of 0.64, which was the highest in the sample.

The Figure 8 also provided the obvious comparable analysis between different frontier markets in terms of the linked degree, darker color means higher correlation. The relatively strong correlations between Morocco (MOR) and Senegal (CEN) (0.24), and Morocco (MOR) and Namibia (NAM) (0.15) suggests market co-movement between these sample markets. These relationships could be driven by policies, including trade agreements, economic collaborations, or external investment flows incentives. However, compared to developed markets, these correlations are still low; it emphasizes the fragmented nature of these sample frontier economies.

On the other hand, some markets exhibited a near-zero or negative correlations, such as Kenya (KEN) and Zambia (ZAM) (-0.01). These weak or negative correlations suggest that their market movements are independent to a certain degree, possibly driven by their different economic structures, monetary policies, and external trade dependencies. These markets are particularly attractive for diversification as they help smooth portfolio returns by reducing exposure to localized risks.

In general, the figures, findings and assumptions, suggest that allocating assets in portfolios across low-correlation frontier markets, can enhance portfolio efficiency and mitigate risk; it is possible to capitalize on the untapped potential and growth of these frontier economies.

4. DISCUSSION

The discussion part presents the empirical results of African frontier markets, focusing on risk-adjusted returns, portfolio diversification, performance dynamics, and comparative outcomes.

4.1. Comparative Analysis of Risk-Adjusted Returns

Figure 9 shows that mean return per unit of risk (MRPUR) ratio for ten frontier markets for the whole period (2015–2024) and two sub-periods (2015–2019 and 2020–2024). The chart provides a comparative analysis of the risk-adjusted returns across different market conditions and investment horizons, and indicates the difference among individual market performance.

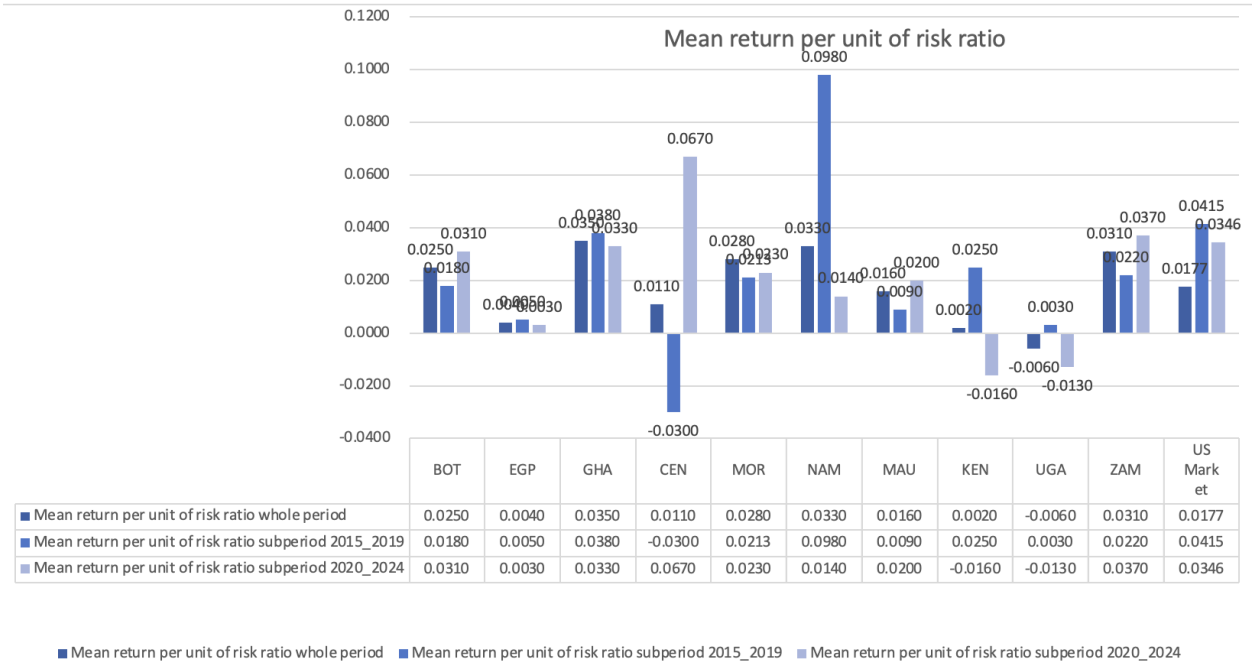


Figure 9. MRPUR periods comparison

Source: Author's work.

4.1.1. Cross-market comparisons by risk-adjusted returns

The figure reveals the significant variation in the MRPUR across different frontier markets. Over the entire period, Ghana (GHA), Namibia (NAM), and Zambia (ZAM) exhibited the highest risk-adjusted returns, suggesting that these markets provided good compensation for market risk. In contrast, Kenya (KEN) and Uganda (UGA) demonstrated negative returns, indicating that investments in these markets carry a higher risk relative to their return potential. Morocco (MOR) and Botswana (BOT) maintained moderate risk-adjusted returns, revealing their stable potential and performance. Additionally, the US market is recognized as a benchmark, with an MRPUR of 0.0177 over the full period. Notably, several frontier markets, including Ghana (0.0350), Namibia (0.0330), and Zambia (0.0310), outperformed the US market in terms of risk-adjusted returns; they illustrate the potential benefits of investing in some of the sample frontier markets.

4.1.2. Sub-Period dynamics of risk-adjusted returns

The Sub-Period MRPUR Analysis 2015–2019: During this period, Namibia (NAM) exhibited a high risk-adjusted return (0.0980), it outperformed other frontier markets significantly. However, Senegal (CEN) recorded a negative MRPUR (-0.0300), suggesting that a significant challenge was faced during this time-frame. Generally, compared with the US market (0.0415), most frontier markets underperformed.

2020–2024: A strong draw-down in market performance was observed in the second sub-period. Senegal (CEN) rebounded strongly, recording the highest risk-adjusted return (0.0670), it suggests improved market conditions and reduced volatility. In contrast, Kenya (KEN) and Uganda (UGA) witnessed a deterioration in performance, with risk-adjusted returns declining to -0.0160 and -0.0130. Namibia (NAM) showed the strongest performance in previous periods, but a significant decline (from 0.0980 to 0.0140) was shown, it was mainly affected by the increased market uncertainty because of Covid-19 (Namibia Statistics Agency, 2020) [16].

4.2. Diversify Portfolio

Throughout the Minitab programming test, it is possible to obtain the results shown in the following chart. Table 3 presents the theoretical results from the Minitab program across different investment

periods. The portfolios are ranked based on their MRPUR in Period 1 (2015–2019), Period 2 (2020–2024), and the whole period (2015–2024).

Table 3. MRPUR

Period	Rank	Portfolio	Mean Return Per Unit of Risk	Key Analysis
Period1	1	MOR + NAM + KEN (Columns 5+6+8)	0.02652	Highest return in a 3-market portfolio. KEN adds marginal value but also brings volatility risk.
Period1	2	MOR + NAM (Columns 5+6)	0.02522	Raised diversification benefits with NAM, increased the MRPUR.
Period1	3	MOR (Column 5)	0.02125	Single-market optimal, highest return per unit of risk. MOR indicated as the core asset.
Period1	4	MOR + NAM + KEN + MAU (Columns 5+6+8+7)	0.01990	Redundant risk from MAU inclusion reduced the efficiency. Over-diversification harms return.
Period1	5	MOR + NAM + KEN + MAU + BOT (Columns 5+6+8+7+1)	0.00759	Severely over-diversification decreased overall returns.
Period2	1	CEN (Column 4)	0.04293	Single-market out-performed; CEN as a short-term high-return asset.
Period2	2	CEN + MOR (Columns 4+5)	0.03388	MOR reduced the portfolio efficiency despite diversification benefits. Prefer standalone CEN in sub-period.
Period2	3	CEN + MOR + BOT (Columns 4+5+1)	0.02939	BOT inclusion decreased returns. Avoid combining volatile markets with CEN.
Period2	4	CEN + MOR + BOT + NAM (Columns 4+5+1+6)	0.01936	NAM increased the downside risk. Negative return trends indicated a poor diversification.
Period2	5	CEN + MOR + BOT + NAM + MAU (Columns 4+5+1+6+7)	0.00999	Extreme over-diversification. MAU adds no value, lead to sub-optimal returns.
Whole Period	1	MOR (Column 5)	0.01573	Long-term stable but has lower returns. MOR remains the only strong asset across periods.
Whole Period	2	MOR + NAM (Columns 5+6)	0.00979	NAM decreased long-term benefits. Avoid combining with MOR for sustained growth.
Whole Period	3	MOR + NAM + CEN (Columns 5+6+4)	0.00743	CEN is failed to improve returns in long-term. Diversification inefficiency evidently in long-term.
Whole Period	4	MOR + NAM + CEN + BOT (Columns 5+6+4+1)	0.00435	Near-zero returns. Adding BOT is an ineffective choice in long-term portfolios.
Whole Period	5	MOR + NAM + CEN + BOT + MAU (Columns 5+6+4+1+7)	-0.00069	Negative returns due to MAU high risk. Avoid this portfolio.

Source: Author's work.

4.2.1. Portfolio performance in period 1 (2015–2019)

During the first period, the MOR + NAM + KEN (Morocco, Namibia, and Kenya) portfolio was the best-performing portfolio, which achieved a high MRPUR at 0.02652. The inclusion of Kenya (KEN) added marginal value, but it increased portfolio volatility. It shows that while diversification improved returns, risk exposure increased synchronously.

The MOR + NAM (Morocco and Namibia) portfolio ranked second but also obtained a high MRPUR at 0.02522. Throughout the 0.15 in correlation value (Figure 8), it is possible to obtain the result that the synergy between Morocco and Namibia proved effective in this period; the strategy reduced market risk while preserving return potential.

A single-market strategy concentrated in Morocco (MOR) was the third optimal portfolio, yielding 0.02125. This finding suggests that Morocco was a strong and core asset during this period; it provided a relatively stable MRPUR compared with other sample markets.

Moreover, over-diversification would lead to diminishing returns. The addition of the sample market of Mauritius (MAU) in MOR + NAM + KEN + MAU reduced the return to 0.0199, indicating that MAU brought additional risk but without proportionate gains. The following ranked strategies further expanded the portfolio to include Botswana (BOT) (MOR + NAM + KEN + MAU + BOT); they reduced the MRPUR to 0.00759. It demonstrated that over-diversification dilutes portfolio efficiency.

4.2.2. Portfolio performance in period 2 (2020–2024)

By the reasons of severe geographical and political volatility that happened like Covid-19 during 2020-2024, the shift affected market dynamics in the second period, where Senegal (CEN) was the best-performing single-market portfolio, which obtained the highest MRPUR of 0.04293. This suggested that Senegal (CEN)'s market nature improved significantly during the period, which allowed it to outperform other diversified portfolios.

The portfolio of CEN + MOR (Senegal and Morocco) produced a lower return of 0.03388; it showed that Morocco did not contribute anything positive to portfolio efficiency in this period. The findings indicate that diversification might dilute superior single-market performance in certain cases.

Then, the inclusion of Botswana (BOT), CEN, and MOR (CEN + MOR + BOT) further decreased returns to 0.02939; it supports the viewpoint that adding volatile markets does not necessarily enhance returns.

Moreover, by expanding the portfolio to CEN + MOR + BOT + NAM (Namibia), it leads to a further decline in MRPUR (0.01936), as Namibia’s presence increased the risk of draw-down and volatility (0.075 at standard deviation, which is the highest among these markets according to Figure 4 and max draw-down from -0.416 to -0.427 according to Figure10. This indicates that not all frontier markets

contribute positively to diversification, and adding high-volatility markets may introduce more risk than reward.

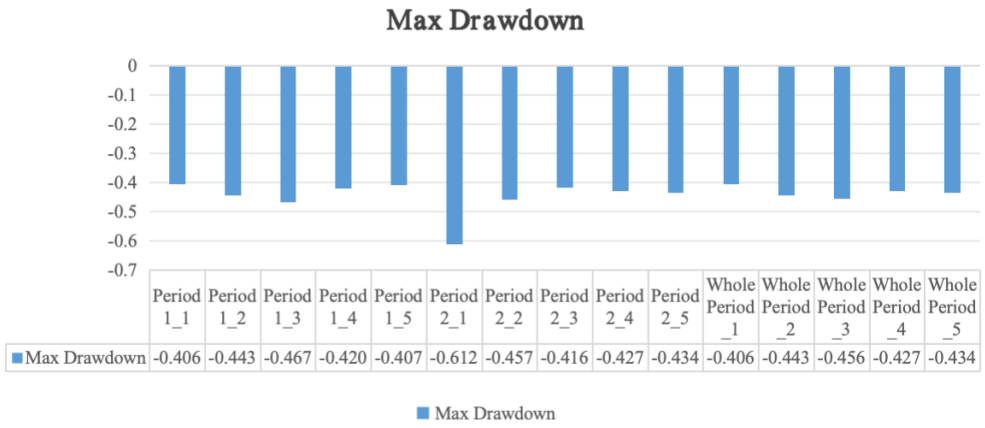


Figure 10. Max drawdown

Source: Author's work.

The worst-performing portfolio in this period was CEN + MOR + BOT + NAM + MAU, which yielded a return of only 0.00999. This extreme over-diversification resulted in significantly reduced efficiency, as MAU failed to add meaningful value.

4.2.3. Portfolio performance over the whole period (2015–2024)

From the perspective of the whole period, Morocco (MOR) emerged as the most stable long-term investment, with an MRPUR of 0.01573. This suggests that while other markets experienced fluctuations, Morocco maintained consistent risk-adjusted performance across different economic cycles through the sample periods.

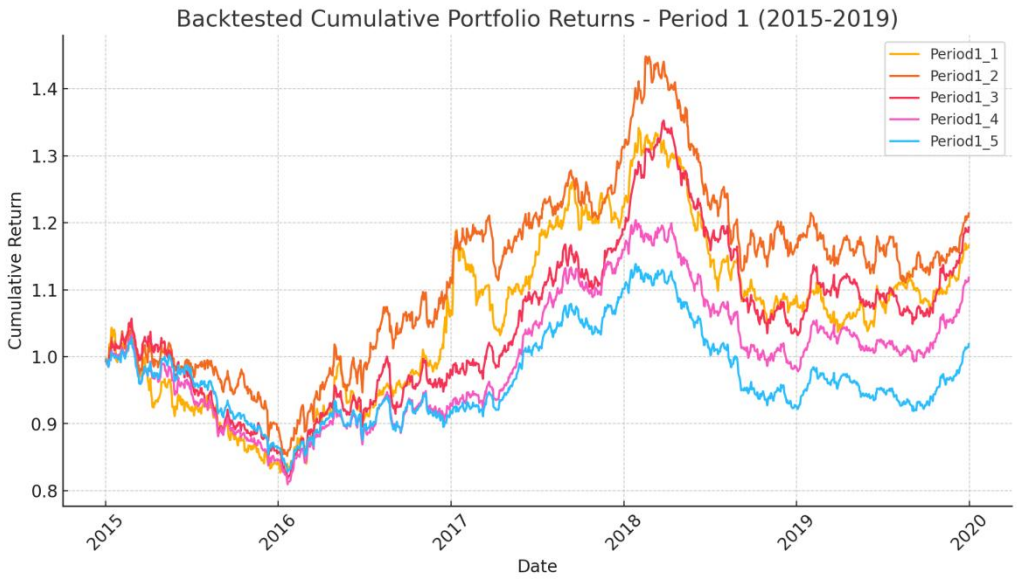
The second-best portfolio MOR + NAM had a lower MRPUR of 0.00979. It reflects the fact that while Namibia initially enhanced diversification, it dragged portfolio performance down in the long term, which made it a less effective portfolio for sustainable growth.

Then, adding CEN (MOR + NAM + CEN); further reduced MRPUR to 0.00743. It illustrates that Senegal's strong short-term performance did not translate into long-term efficiency.

Moreover, a more diversified portfolio, MOR + NAM + CEN + BOT, had an MRPUR that approached zero (0.00435); it shows that Botswana's inclusion led to over-diversification and reduced the MRPUR.

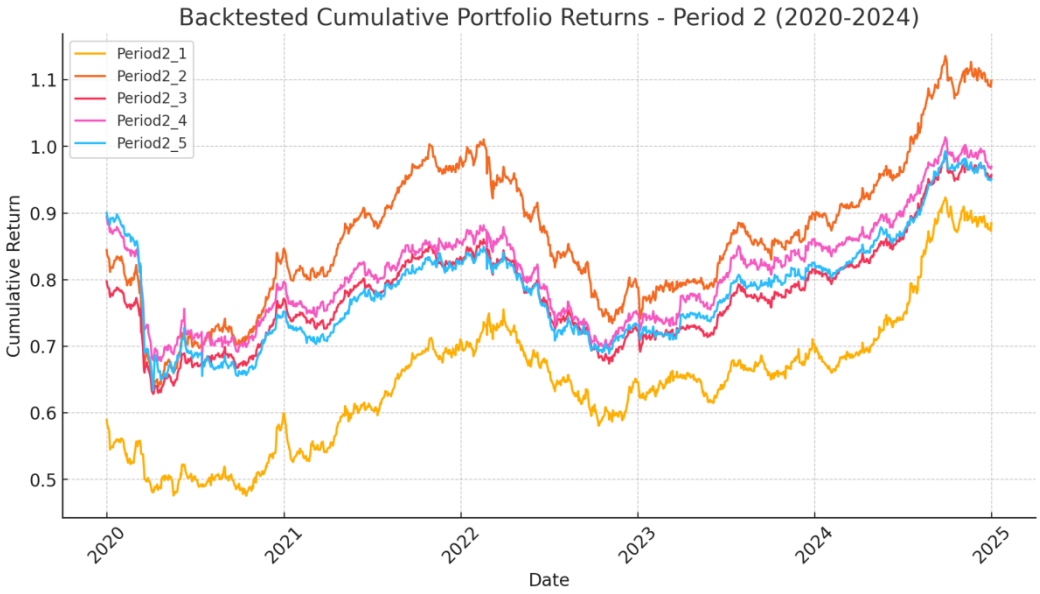
The worst-performing portfolio for the entire period was MOR + NAM + CEN + BOT + MAU, which had a negative return of -0.00069. The poor performance of MAU, which combined with extreme diversification, made this portfolio highly inefficient through the whole period.

Figures 11, 12, and 13 present the statistical back-tested cumulative return performance of different portfolio compositions from 2015 to 2024 which were based on Python. These figures compared theoretical portfolio optimization results from Minitab with realized performance in the markets.



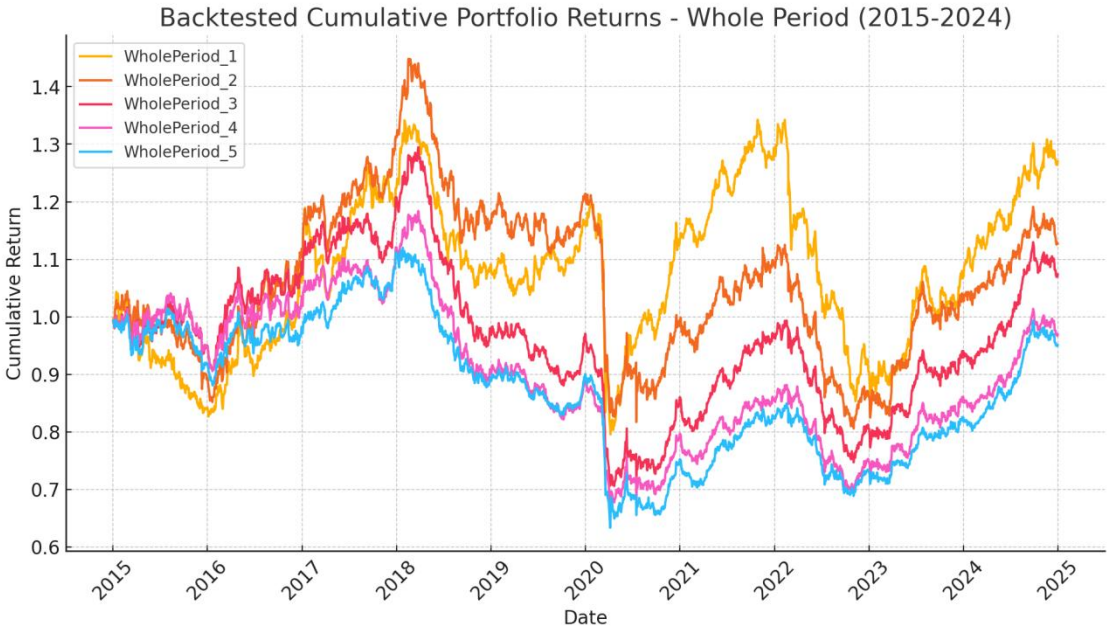
Figures 11. Back-tested cumulative portfolio returns in period 1

Source: Author's work.



Figures 12. Back-tested cumulative portfolio returns in period 2

Source: Author's work.



Figures 13. Back-tested cumulative portfolio returns in whole period

Source: Author's work.

The general results over time could be depicted in figures 11, 12, 13, which demonstrates that Morocco has maintained its stability for an extended period, Namibia could be a good choice for short-term, and Senegal has rebounded from 2020 to 2024. In general, the findings indicate that selective diversified portfolio enhanced the efficiency of investments, while an excessive amount of diversification had negative effect in returns. Therefore, Morocco is consistently the most advantageous frontier market asset for long-term investment choice.

5. CONCLUSION

This study has explored the diversification benefits of investing in a sample of African frontier markets. The findings indicated that while these markets offered benefits because of their lower integration with developed markets, they also brought unique challenges, which included political instability, regulatory inefficiencies, and market liquidity. By examining key performance variables like mean return per unit of risk (MRPUR), correlation, and portfolio optimization strategies back-tests, the research provided an empirical analysis into the inclusion of frontier markets in global investment portfolios.

The empirical results demonstrated that Morocco and Namibia in the period 2015-2019 and Senegal (CEN) in the period 2020-2024 exhibited moderate risk levels while delivering consistent returns, which made them attractive options for long-term investors.

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