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# The Risk-retern Tradeoff among Equity Factors: Evidence from A-share Market

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## ABSTRACT

Based on a time-series analysis of equity factors, this study documents a significant positive risk-return tradeoff for size and value factors in A-share Market, consistent with the predictions of the Arbitrage Pricing Theory (APT). This relationship remains robust after controlling for covariance with the market factor, providing support for Merton's Intertemporal CAPM (ICAPM). In contrast, the risk-return tradeoff is found to be insignificant for the market factor itself, as well as for other factors. The conclusion continues to hold across a battery of robustness checks. This study refines the theoretical understanding of dynamic asset pricing by empirically validating that the intertemporal risk-return relationship.

## KEYWORDS

Risk-Return Tradeoff; Equity Factors; Realized variance

## 1. INTRODUCTION

According to the conditional Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) [1] and Lintner (1965) [2], a positive risk–return tradeoff should prevail at the aggregate level—that is, higher (conditional) expected market returns should be associated with higher (conditional) market variance. This central prediction has spurred a large body of empirical research. Numerous studies, employing a variety of methodologies, have reported evidence consistent with such a positive relationship (e.g., Bollerslev et al., 1988 [3]; Scruggs, 1998 [4]; Ghysels et al., 2005 [5]; Pástor et al., 2008 [6]; Hedegaard & Hodrick, 2016 [7]). In contrast, another strand of the literature has documented a negative relation between risk and return (e.g., Glosten et al., 1993 [8]; Whitelaw, 1994 [9]; among others).

However, the vast majority of this work has concentrated on the aggregate stock market, leaving the risk–return dynamics of finer market segments relatively unexplored.[2] Our study helps address this gap by investigating the risk–return tradeoff within several widely studied zero-cost equity factors from the empirical asset pricing literature.

In line with the theoretical foundation of the market risk–return tradeoff under the conditional CAPM, the risk–return relationship for individual factors can be motivated by conditional single-factor models, which empirically extend the Arbitrage Pricing Theory (APT) of Ross (1976) [10]. Alternatively, this relationship can also be justified within conditional two-factor models, drawing on the Intertemporal CAPM (ICAPM) framework of Merton (1973) [11].

In this study, we adopt both conditional single-factor and two-factor models—where the latter include the market factor along with each of the non-market factors from the multifactor models of Carhart (1997) [12] and Fama and French (2015, 2016) [13, 14]—to provide theoretical grounding for our empirical analysis. These models imply a positive risk–return tradeoff for the size, value, momentum, profitability, and investment factors: that is, each factor's risk premium should be positively related to its own conditional variance, whether or not we control for its conditional covariance with the market.

To operationalize these concepts, we proxy the unobserved conditional variances and covariances using monthly realized measures constructed from daily factor returns. This approach is well established in the literature, with precedents in works such as , Goyal and Santa-Clara (2003) [15], Bali et al. (2005) [16], Guo et al. (2009) [17], Barroso and Santa-Clara (2015) [18], among others.

By using monthly data from 1995 to 2025, we first conducts univariate regressions to examine the relationship between factor returns and lagged variance. The results indicate a positive in-sample risk-return relationship for the size (SMB) and value (HML) factors, whose risks help predict future returns with economic significance. This finding aligns with the theoretical predictions of the conditional single-factor APT. Notably, no such relationship is observed for the other factors.

By incorporating the realized covariance between the market factor and other factors into the baseline regression, bivariate regressions are performed to further investigate the relationship between returns and both factor-specific risk and market-linked risk. For the size (SMB) and value (HML) factors, the coefficients of realized variance remain significantly positive, while the covariance with the market shows weak statistical significance. This suggests that factor returns are primarily driven by their own volatility, with market connectivity playing a secondary role.

The robustness of the results is verified through multiple approaches. Using Bootstrap simulation-based statistical inference, the positive risk-return relationship for the size (SMB) and value (HML) factors remains highly significant. By including an interaction term between the business cycle dummy and lagged variance, the size factor (SMB) exhibits a significantly positive risk-return relationship during both expansion and contraction periods, with no statistically significant difference across cycles. Similar evidence is found when using the size factor proposed by Hou et al. (2015) [19] and EP factor.

Finally, the predictive power of factor realized variance for future factor returns is examined in multivariate regression with size (SMB) and value (HML) factors and their covariance.

By analyzing the risk-return relationship of non-market factors, this study provides a theoretical basis and practical guidance for constructing more rigorous multi-factor asset pricing models. The significantly positive risk-return relationship of the size (SMB) and value (HML) factors, even after controlling for the market factor, indicates that their returns stem from compensation for non-systematic risk. Their risk premiums can be predicted through volatility measures (e.g., realized variance) and validated economically, justifying their inclusion in multi-factor models. In contrast, other factors, such as momentum (UMD), fail to exhibit a robust positive risk-return relationship, suggesting that their returns may originate from market inefficiencies (e.g., behavioral biases or short-term arbitrage) rather than systematic risk compensation. Traditional multi-factor models (e.g., the Fama-French five-factor model) often rely on empirical discoveries for factor selection. Validating the risk-return relationship of factors helps distinguish whether returns are derived from risk compensation or market frictions in the "factor zoo," thereby enriching the theoretical foundation of factors. Meanwhile, by testing the statistical and economic significance of the risk-return relationship, this study offers stricter criteria for factor selection, making the models more aligned with the theoretical framework of the Intertemporal Capital Asset Pricing Model (ICAPM).

This study is directly related to the literature on time-series risk-return trade-off. Guo et al. (2009) [17] find a positive relationship between the value premium (HML) and its conditional variance, indicating that value stocks carry higher systematic risk compared to growth stocks. Unlike Guo et al. (2009) [17], this study employs more advanced second-moment estimation methods instead of traditional GARCH-M models. Moreover, it validates the in-sample and out-of-sample risk-return relationships for multiple factors proposed by Fama and French (2015, 2016) [13, 14] and Carhart

(1997) [12]. The analytical framework follows Barroso and Maio (2024) [20]. Although existing research has applied this framework to developed markets, no study has systematically tested its applicability in the A-share market. While Barroso and Maio (2024) [20] find significant risk-return relationships for the investment and profitability factors in developed markets, this study identifies significant positive risk-return relationships only for the size and value factors in the A-share market. This reflects differences in risk compensation mechanisms across varying economic structures and market efficiency gradients.

The study is structured as follows: Section 2 establishes the theoretical foundation for the factor risk-return trade-off, arguing the theoretical link between conditional variance and expected returns. Section 3 details data sources and variable construction methods. Section 4 empirically tests the risk-return relationships of various factors in the stock market using time-series regression methods, with a focus on robustness checks for the size (SMB) and value (HML) factors. Section 5 summarizes the main findings.

## 2. THEORETICAL FRAMEWORK

In this section, we establish the theoretical foundation for the subsequent empirical analysis, drawing on both the Arbitrage Pricing Theory (APT) by Ross (1976) and the Intertemporal Capital Asset Pricing Model (ICAPM) by Merton (1973).

### 2.1. APT

Consider a linear single-factor model in which the risk factor is represented by the return on a zero-cost equity portfolio (F), constituting an excess return. This model serves as an empirical implementation of the APT framework (Ross, 1976) [10], implying that the traded factor captures a substantial portion of the time-series variation in the cross-section of realized stock returns (e.g., Cooper et al., 2021 [21]).

According to Barroso and Maio (2024) [20], the conditional expected return-covariance equation is given by:

$$E_t(R^e_{i,t+1}) \approx \gamma \mathrm{cov}_t(R^e_{i,t+1}, F_{t+1}) \quad (1)$$

Where $R^e_i$ denotes the excess return on risky asset $\mathrm{i}$, $E_t(.)$ is the conditional expectation at time $\mathrm{t}$, and $\mathrm{cov}_t(.,.)$ is the conditional covariance at time $\mathrm{t}$. Here, $\mathrm{F}$ is a traded risk factor (i.e., an excess return), and γ represents the conditional covariance risk price, assumed to be constant over time.

Because the factor is traded, the same pricing relation applies to the factor itself (e.g., Cochrane, 2005 [22]; Lewellen et al., 2010 [23]):

$$E_t(F_{t+1}) \approx \gamma \mathrm{var}_t(F_{t+1}) \quad (2)$$

Denote an empirical proxy for the unobserved conditional variance of the factor. The risk-return tradeoff can be empirically tested via the predictive regression:

$$F_{t+1} = \alpha_F + \theta_F \underset{t}{\widehat{\mathrm{var}}}(F_{t+1}) + \varepsilon_{F,t+1} \quad (3)$$

Where $\varepsilon_F$ is a zero-mean forecast error. Taking conditional expectations yields:

$$E_t(F_{t+1}) = \alpha_F + \theta_F \underset{t}{\widehat{\mathrm{var}}}(F_{t+1}) \quad (4)$$

This formulation indicates that the estimated intercepts $\alpha_F$ in the forecasting regressions correspond to pricing errors for each factor. If the single-factor APT is correctly specified (i.e., with zero pricing errors), the intercept estimates should not be statistically different from zero. However, potential misspecification—such as omission of relevant risk factors—may lead to non-zero intercepts.

Alternatively, imposing a zero intercept may yield a more powerful test of the time-series risk-return tradeoff. By restricting the pricing error to zero, we can directly examine whether a positive association exists between factor risk (as measured by conditional variance) and expected return.

In the following sections, F refers to one of the zero-cost portfolios: size (SMB), value (HML), profitability (RMW), or investment (CMA), momentum (UMD). Each single-factor model thus represents a restricted version of the multifactor models proposed by Fama and French (1993, 2015, 2016) [13, 14] and Carhart (1997) [12].

For example, the forecasting regression for the momentum factor is specified as:

$$UMD_{t+1} = \alpha_{UMD} + \theta_{UMD} \underset{t}{\widehat{\text{var}}}(UMD_{t+1}) + \varepsilon_{UMD,t+1} \tag{5}$$

With analogous regressions applied to each of the other factors. Since each equity factor is constructed to yield a positive risk premium, the slope estimates $\theta$ from these predictive regressions are expected to be positive.

## 2.2. ICAPM

We now consider a two-factor linear asset pricing model that includes the excess market return (MKT) and the return on a zero-cost equity portfolio (F). This setup can be viewed as an empirical implementation of Merton's (1973) [11] Intertemporal CAPM (ICAPM), in which F serves as a "hedging" factor that helps offset future shifts in the investment opportunity set (e.g., Maio & Santa-Clara, 2012 [15]; Cooper & Maio, 2019a [24]). Prior studies show that many commonly used equity factors satisfy sign restrictions consistent with the ICAPM. That is, state variables linked to these factors predict future market returns, market volatility, or economic activity in a manner consistent with positive risk prices. This supports the use of two-factor models as valid empirical representations of the ICAPM.

the ICAPM implies the following conditional covariance pricing equation:

$$E_t(R^e_{i,t+1}) \approx -b_{MKT}\text{cov}_t(R^e_{i,t+1}, MKT_{t+1}) - b_F\text{cov}_t(R^e_{i,t+1}, F_{t+1}) \tag{6}$$

Where $b_j < 0 (j = MKT, F)$ are coefficients of the stochastic discount factor (SDF). If $F$ is a traded factor, the pricing equation also applies to itself:

$$E_t(F_{t+1}) \approx -b_{MKT}\text{cov}_t(F_{t+1}, MKT_{t+1}) - b_F\text{var}_t(F_{t+1}) \tag{7}$$

These expressions highlight the risk–return tradeoff for the hedging factor. Since $b_F < 0$, higher factor risk—as measured by its conditional variance—implies a higher conditional risk premium. This reflects the idea that a positive shock in the hedging factor signals improved investment opportunities ("good times"), thereby reducing the marginal value of multiperiod wealth (i.e., the SDF). Similarly, an increase in the conditional covariance between F and the market also raises the factor risk premium, as $b_{MKT} < 0$. Here, $-b_{MKT}$ can be interpreted as the average relative risk aversion in the economy.

To test the conditional ICAPM empirically, we estimate the following predictive regression:

$$F_{t+1} = \alpha_F + \theta_{F,MKT}\widehat{\text{cov}}_t(F_{t+1}, MKT_{t+1}) + \theta_F\widehat{\text{var}}_t(F_{t+1}) + \varepsilon_{F,t+1} \tag{8}$$

Where $\widehat{\text{cov}}_t(.,.)$ denotes an empirical proxy for the conditional covariance, and $\varepsilon_F$ is a zero-mean forecast error. Given the identities $\theta_{F,MKT} \equiv -\text{b}_{\text{MKT}}$ and $\theta_F \equiv -\text{b}_{\text{F}}$, both slope coefficients are expected to be positive, consistent with positive risk premia for all equity factors considered.

If a conditional version of the CAPM (Sharpe, 1964 [1]; Lintner, 1965 [2]) holds, the market risk–return tradeoff is given by:

$$E_t(MKT_{t+1}) = -b_{MKT}\text{var}_t(MKT_{t+1}) \tag{9}$$

Which can be estimated via:

$$MKT_{t+1} = \alpha_{MKT} + \theta_{MKT}\widehat{\text{var}}_t(MKT_{t+1}) + \varepsilon_{MKT,t+1} \tag{10}$$

Where $\theta_{MKT} \equiv -\text{b}_{\text{MKT}}$ provides an estimate of the risk aversion coefficient. This relationship lies at the heart of the extensive empirical literature on the aggregate risk–return tradeoff.

It is important to note that the bivariate forecasting regression above serves as a test of the two-factor ICAPM, incorporating only one non-market factor (e.g., SMB) alongside the market. We do not aim to test a full conditional multifactor model (e.g., Carhart four-factor model) designed for cross-sectional pricing. Instead, our goal is to evaluate the time-series risk–return tradeoff of individual equity factors within a parsimonious ICAPM setting—one that includes the market and a single hedging factor. This approach allows us to directly compare the strength of the risk–return relation within market segments to that of the aggregate market, using the conditional CAPM as a natural benchmark. The two-factor ICAPM represents the closest extension to the baseline conditional CAPM, making it well suited for this purpose.

## 3. DATA AND VARIABLES

To estimate the risk-return trade-off relationship of various stock factors, it is necessary to construct empirical proxy variables for the unobservable conditional factor variances. Following mainstream research approaches (such as Goyal & Santa-Clara, 2003 [15]; Bali et al., 2005 [16]; Barroso et al., 2025 [25]), we employ the realized variance of the current month. Given that few months in the A-share market have more than 21 actual trading days, we refer to the method of Bali et al. (2005) [16] and use all available daily return data within the month to construct the realized variance. The calculation formula is as follows:

$$RV_t = \sum_{n=1}^{N} r_{t,n}^2 \tag{11}$$

$\text{r}_{\text{t,n}}$ represents the factor return on the $\text{n}$ trading day in month $\text{t}$, and $\text{N}$ denotes the total number of trading days in that month. Samples failing to meet the criterion $\text{N} \geq 15$ are directly excluded. A total of 8 monthly samples were excluded during the sample period of this study. This estimator serves as the conditional variance estimator for a specific factor in month $\text{t}$ (i.e., $\widehat{\text{var}}_{\text{t}}(\text{F}_{\text{t+1}})$). Both daily and monthly factor data were obtained from the CSMAR database, covering the sample period from 1995 to 2023.

**Table 1.** Descriptive statistics for realized variances

| | Mean (%) | Sd (%) | Min (%) | Max (%) |
|---|---|---|---|---|
| MKT_RV | 0.554 | 0.940 | 0.022 | 14.262 |
| SMB_RV | 0.134 | 0.180 | 0.003 | 2.349 |
| HML_RV | 0.130 | 0.152 | 0.003 | 0.968 |
| RMW_RV | 0.095 | 0.111 | 0.007 | 0.837 |
| CMA_RV | 0.046 | 0.054 | 0.003 | 0.429 |
| UMD_RV | 0.125 | 0.199 | 0.007 | 2.395 |
| This table reports descriptive statistics for the realized variance of each factor. MKT, SMB, HML, RMW, CMA and UMD denotes Fama-French -Carhart market, size, value-growth, momentum, profitability, and investment factor, respectively. The sample is 1995.01-2025.03. | | | | |

The descriptive statistics of the realized variances of various factors in Table 1 reveal the following insights: The market factor (MKT) has the highest average realized variance, confirming its role as the primary source of systematic risk. The size (SMB) and value (HML) factors show similar risk levels (~24% of market risk), followed by profitability (RMW) and momentum (UMD). The investment factor (CMA) has the lowest volatility (8.3% of market risk).MKT variance also exhibits the highest standard deviation, indicating instability across periods. SMB and HML variances are relatively stable, with CMA being the most stable. The market factor (MKT) shows an exceptionally high maximum value, indicating extreme volatility during certain periods within the sample. By contrast, the size factor (SMB) reaches a peak of 2.349%, exceeding other style factors and pointing to pronounced fluctuations in small-cap returns over specific intervals. Meanwhile, the investment factor (CMA) displays the lowest maximum volatility (0.429%), reinforcing its profile as a comparatively stable strategy.

**Table 2.** Descriptive Statistics of Factor Returns

| | Mean (%) | Sd (%) | Skewness | Kurtosis | Min (%) | Max (%) |
|---|---|---|---|---|---|---|
| MKT | 1.002 | 7.828 | 0.380 | 4.85 | -24.960 | 33.398 |
| SMB | 0.624 | 4.702 | -0.149 | 5.49 | -23.145 | 20.364 |
| HML | 0.481 | 4.686 | 1.034 | 8.93 | -17.599 | 28.113 |
| RMW | 0.175 | 3.862 | 0.179 | 5.79 | -15.405 | 17.845 |
| CMA | -0.157 | 2.729 | -0.148 | 6.81 | -12.841 | 14.225 |
| UMD | -0.038 | 4.250 | -0.171 | 4.33 | -15.999 | 13.671 |
| This table reports descriptive statistics for the returns of each factor. MKT, SMB, HML, RMW, CMA and UMD denotes Fama-French-Carhart market, size, value-growth, momentum, profitability, and investment factor, respectively. The sample is 1995.01-2025.03. | | | | | | |

The descriptive statistics of factor returns in Table 2 show that the market factor (MKT) offers the highest absolute return but also represents the largest source of risk. The size (SMB) and value (HML) factors provide significant positive premiums and are the main contributors to excess returns. In contrast, the profitability factor (RMW) exhibits a weak premium, while the investment (CMA) and momentum (UMD) factors show negative average returns. Notably, the value factor (HML) displays strong right skew, high kurtosis, and heavy-tailed characteristics, suggesting that its returns may be driven by a few extreme positive outcomes.

**Table 3.** AR(1) Estimates for Factor Realized Variances

| | ∅ | P-value | $R^2$(%) |
|---|---|---|---|
| MKT | $0.234^{***}$ | (0.05) | 5 |
| SMB | $0.331^{***}$ | (0.05) | 11 |
| HML | $0.675^{***}$ | (0.04) | 46 |
| RMW | $0.438^{***}$ | (0.05) | 19 |
| CMA | $0.627^{***}$ | (0.04) | 39 |
| UMD | $0.301^{***}$ | (0.05) | 9 |
| This table presents the AR(1) coefficient estimates for the realized variance of each factor. P-value are reported in parentheses. * p < 0.10, ** p < 0.05, *** p < 0.01. | | | |

Table 3 presents the AR(1) estimation results of the realized variance for all factors. The volatility of the value (HML), investment (CMA), and profitability (RMW) factors exhibits high persistence, with AR(1) coefficients all exceeding 0.4. In contrast, the market (MKT) and momentum (UMD) factors show relatively weaker volatility persistence. The size factor (SMB) demonstrates a moderate level of persistence. All estimated coefficients are highly statistically significant.

Additionally, we computes the Sharpe ratio, maximum drawdown, and cross-factor correlations. The results indicate that the size (SMB) and market (MKT) factors perform best, with Sharpe ratios of 0.460 and 0.443, respectively. The value factor (HML) also delivers positive returns, whereas the profitability factor (RMW) offers only a marginal premium. The investment (CMA) and momentum (UMD) factors exhibit negative Sharpe ratios. The market factor (MKT) experiences the deepest drawdown (–66.73%), while the investment factor (CMA), despite its lowest volatility, undergoes the second-largest decline (–62.39%), highlighting its extreme tail risk. The profitability factor (RMW) shows the strongest defensive characteristics with the shallowest drawdown (–35.25%).

Regarding correlations, all factors exhibit near-zero correlation with the market factor except for a significant weak negative correlation between profitability (RMW) and market (MKT), confirming that long-short factor returns should originate from idiosyncratic risks rather than market exposure. Notably, a strong negative correlation (–0.639*) exists between the size (SMB) and profitability (RMW) factors.

## 4. ESTIMATING THE RISK-RETURN TRADEOFF

In this section, guided by theoretical foundations, we examine the predictive power of realized variance for future returns and perform robustness checks.

### 4.1. Univariate Regression

First, we directly examine the risk-return relationship for each factor by estimating the following univariate model using OLS:

$$F_{t+1} = \alpha + \theta RV_t + \varepsilon_{t+1} \tag{12}$$

Following the approach of Barroso and Maio (2024) [20], we estimate this model both with and without an intercept. Heteroskedasticity-robust t-statistics are used, and both two-tailed and one-tailed p-values are reported to evaluate significance, as theory predicts the slope coefficient θ should be positive.

The results of the risk-return relationship for each factor are presented in Table 4. In the model with an intercept, the risk-return relationship for the market factor (MKT) is negative, consistent with prior studies (Glosten et al. 1993 [8], Whitelaw 1994 [9], among others), though the absolute t-statistic is small and not statistically significant. The investment factor (CMA) also shows a negative risk-return relationship, but with a very small and insignificant t-statistic. The momentum factor (UMD) exhibits a slope close to zero, a t-statistic near zero, and an $R^2$ of zero, indicating almost no linear relationship between risk and return for this factor.

The profitability factor (RMW) has a positive slope, but the coefficient is not significant, and the model exhibits very low explanatory power, suggesting that the volatility of the profitability factor does not predict its future returns.

The size factor (SMB) shows a significantly positive risk-return relationship at the 1% level. This provides strong evidence that higher risk in the size factor is associated with higher future returns, supporting the classical theory that increased volatility in small-cap stocks predicts higher future excess returns. The insignificant intercept suggests no pricing error in the model. According to the regression results, a one-standard-deviation increase in the realized variance of SMB predicts an increase in next month's return of approximately 4.134×0.18%≈0.74%

The value factor (HML) also shows a significantly positive risk-return relationship at the 10% level, with an insignificant intercept, consistent with the intuition of "high risk, high return." The $R^2$ values for both SMB and HML are substantially higher than those of other factors. A one-standard-deviation increase in HML volatility predicts a return increase of approximately 4.801×0.152%≈0.73%, indicating that realized variance has economically meaningful predictive power for the returns of the size and value factors. These results remain consistent when one-tailed p-values are used to assess significance.

**Table 4.** Risk-return tradeoff: univariate regression

| Panel A: With intercept | | | | | |
|---|---|---|---|---|---|
| | α(%) | t | θ | t | $R^2$(%) |
| MKT | 1.15 | 2.73(***) | -0.196 | -0.42 | 0.1 |
| SMB | 0.08 | 0.28 | 4.134 | 3.2(***) | 2.5 |
| HML | -0.15 | -0.46 | 4.801 | 1.68(*) | 2.4 |
| RMW | -0.01 | -0.05 | 1.879 | 0.67 | 0.3 |
| CMA | -0.08 | -0.46 | -1.489 | -0.40 | 0.1 |
| UMD | -0.04 | -0.16 | 0.014 | 0.01 | 0 |
| Panel B: Without intercept | | | | | |
| MKT | | | 0.341 | 0.58 | 0.2 |
| SMB | | | 4.334 | 3.65(***) | 4.2 |
| HML | | | 4.321 | 1.98(**) | 3.4 |
| RMW | | | 1.814 | 0.85 | 0.5 |
| CMA | | | -2.188 | -0.75 | 0.3 |
| UMD | | | -0.076 | -0.06 | 0 |

This table reports the results for regressions of each factor return on its lagged realized variance. MKT, SMB, HML, RMW, CMA and UMD denotes Fama-French -Carhart market, size, value-growth, momentum, profitability, and investment factor, respectively. denotes the intercept estimate, while denotes the slope estimate. represents corresponding -ratio. is the coefficient of determination. A -ratio marked with *, **, and *** denotes statistical significance at the 10%, 5%, and 1% level, respectively. In Panel B, the regressions without the intercept. The sample is 1995.01-2025.03.

In the model without an intercept, the significantly positive risk-return relationships for the size (SMB) and value (HML) factors remain, and their significance levels improve. A one-standard-deviation increase in SMB realized variance predicts a return increase of approximately 4.334×0.18%≈0.78%, while a one-standard-deviation increase in HML volatility predicts a return increase of approximately

4.321×0.152%≈0.66%. The results for other factors in the model without an intercept are consistent with those in Panel A and remain statistically insignificant.

The results in Table 4 indicate that the size (SMB) and value (HML) factors exhibit a positive risk-return relationship. The slope coefficients in the univariate regressions for these two factors satisfy the sign restrictions implied by the conditional single-factor APT. In contrast, the results for the remaining factors—whether showing negative risk-return relationships or statistical insignificance—are inconsistent with theoretical priors.

### 4.2. Bivariate Regression

To examine the predictive role of both factor-specific risk and market-linked risk on returns, we extend the univariate regression model by incorporating the realized covariance between factor returns and market returns, constructing the following bivariate regression model:

$$R_{t+1} = \alpha + \theta_1 RV_t + \theta_2 RCM_t + \varepsilon_{t+1} \tag{13}$$

$$RCM_t = \sum_{n=0}^{N} F_{d_{t-n}} \cdot MKT_{d_{t-n}} \tag{14}$$

Where $RCM_t$ denotes the realized covariance between the factor return and the market return in month t. This model is designed to test the ICAPM theory discussed in Section 2. It allows us to evaluate whether a positive factor risk-return relationship persists after controlling for covariance with the market factor. Following the approach used in the univariate analysis, we estimate the model both with and without an intercept.

The bivariate regression results are reported in Table 5. Under both specifications (with and without intercept), the estimated coefficient of $RCM_t\,(\theta_2)$ is statistically insignificant, providing little evidence that covariance risk with the market is priced or commands a positive return.

The coefficient of realized variance $RV_t$ $(\theta_1)$ for the size factor (SMB) remains significantly positive, indicating a strong positive risk-return relationship even after controlling for covariance with the market. For the value factor (HML), $\theta_1$ is marginally significant in the model with an intercept and becomes significant in the model without an intercept.

A one-standard-deviation increase in the realized variance of SMB predicts an increase in next month’s return of approximately 4.735×0.18%≈0.85%. Similarly, a one-standard-deviation increase in the realized variance of HML predicts a return increase of approximately 4.282×0.152% ≈0.65%.

For most other non-market factors, $\theta_1$ are positive, though the coefficients remain statistically insignificant.

**Table 5.** Risk-return tradeoff: bivariate regression

| Panel A: With intercept | | | | | | | |
|---|---|---|---|---|---|---|---|
| | α(%) | t | $\theta_1$ | t | $\theta_2$ | t | $R^2$(%) |
| SMB | 0.11 | 0.28 | 4.453 | 3.3(***) | -2.176 | -0.84 | 2.8 |
| HML | -0.14 | -0.43 | 4.751 | 1.63 | 0.294 | 0.14 | 2.4 |
| RMW | -0.02 | -0.08 | 1.821 | 0.59 | -0.255 | -0.10 | 0.3 |
| CMA | -0.08 | -0.48 | -1.269 | -0.32 | -0.744 | -0.27 | 0.1 |
| UMD | -0.04 | -0.14 | 0.0007 | 0 | 0.096 | 0.04 | 0 |
| Panel B: Without intercept | | | | | | | |
| SMB | | | 4.735 | 3.53(***) | -2.070 | -0.82 | 4.4 |
| HML | | | 4.282 | 1.95(*) | 0.385 | 0.19 | 3.4 |
| RMW | | | 1.736 | 0.67 | -0.231 | -0.09 | 0.5 |
| CMA | | | -2.027 | -0.64 | -0.683 | -0.25 | 0.4 |
| UMD | | | -0.087 | -0.07 | 0.130 | 0.06 | 0 |

This table reports the results for regressions of each factor return on its lagged realized variance (captured by $\theta_1$) and lagged realized covariance with market factor (captured by $\theta_2$). SMB, HML, RMW, CMA and UMD denotes Fama-French-Carhart market, size, value-growth, momentum, profitability, and investment factor, respectively. $\alpha$ denotes the intercept estimate, $t$ represents corresponding $t$-ratio. $R^2$ is the coefficient of determination. A $t$-ratio marked with *, **, and *** denotes statistical significance at the 10%, 5%, and 1% level, respectively. In Panel B, the regressions without the intercept. The sample is 1995.01-2025.03.

In summary, the empirical results for the size (SMB) and value (HML) factors demonstrate strong consistency with the two-factor ICAPM framework. Even after isolating the influence of the market, both SMB and HML continue to exhibit a positive risk-return relationship. The results further indicate that, compared to their own factor variance, the realized covariance with the market factor plays a secondary role, as evidenced by its weak statistical significance.

## 4.3. Robustness Check

In this subsection, we conduct a series of robustness checks on the univariate regression analysis. Based on the earlier findings, these tests focus exclusively on the size (SMB) and value (HML) factors.

### 4.3.1. Bootstrap simulation.

To robustly assess the asymptotic t-statistics, we implement a bootstrap simulation to compute pseudo t-statistics. This approach generates an empirical distribution of the estimated regression slope, providing a more accurate approximation of its finite-sample behavior. The simulation involves resampling both factor returns and predictor variables with 10,000 repetitions, under the null hypothesis that factor returns are unpredictable and the predictor follows a AR(1) process:

$$F_{t+1} = \mu + u_{t+1} \tag{15}$$

$$RV_{t+1} = \psi + \phi RV_t + e_{t+1} \tag{16}$$

The procedure explicitly accounts for the high persistence in factor realized variance and controls for correlation between regression residuals and the predictor, thereby effectively correcting for Stambaugh (1999) bias. The pseudo t-statistic is derived from bootstrap standard errors, reflecting the variability of slope estimates across the simulated samples.

Empirical results show that, in the regression specification without an intercept, the pseudo t-statistics for the slope coefficients of the size (SMB) and value (HML) factors are 3.65 and 1.98, respectively, both significant at conventional confidence levels. This reinforces the inferences based on asymptotic theory. Furthermore, the estimated intercepts for both factors remain statistically insignificant, supporting the appropriateness of the no intercept model specification.

### 4.3.2. Business cycle analysis.

Existing research suggests that factor premiums exhibit a countercyclical nature (Cooper and Maio 2019b [28]). To investigate whether the risk-return relationship varies over the business cycle, we extend the baseline regression by incorporating an interaction term between factor realized variance and a business cycle dummy variable. The model is specified as:

$$F_{t+1} = \theta_1 \cdot \text{Cycle}_t \cdot RV_t + \theta_2 \cdot (1 - \text{Cycle}_t) \cdot RV_t + \varepsilon_{t+1} \tag{17}$$

The business cycle dummy is constructed using the Hodrick-Prescott (HP) filter. The HP filter minimizes the following objective function:

$$\min_{\{s_t\}} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} [(s_{t+1} - s_t) - (s_t - s_{t-1})]^2 \right\} \tag{18}$$

Where $y_t$ denotes the annual real GDP growth rate published by the National Bureau of Statistics, and $s_t$ represents its long-term trend. The cyclical component is obtained as $y_t - s_t$. We define $\text{Cycle}_t = 1$ when the cyclical component is positive (expansion phase) and $\text{Cycle}_t = 0$ when it is negative (contraction phase). Following Ravn et al. (2002) [26], the smoothing parameter λis set to 6.25.

The results show that the SMB factor exhibits a significantly positive risk-return relationship during both contractions (θ=4.726) and expansions (θ=4.065). Although the point estimate is larger during contractions, the difference across cycles is not statistically significant. Similarly, the HML factor shows positive risk-return coefficients in both contraction (θ=3.175) and expansion (θ=2.565)phases, consistent with theoretical expectations. However, the coefficient during contractions is not significant, and the cyclical difference remains statistically insignificant.

These findings indicate that the risk-return relationships of the size (SMB) and value (HML) factors are largely stable across different phases of the business cycle.

### 4.3.3. Alternative factors.

To assess the robustness of the univariate model, we replace the conventional size (SMB) and value (HML) factors with alternative proxies that share similar economic interpretations but differ in construction. For the size factor, we use the ME factor from Hou et al. (2015) [19], while for the value factor, we employ the earnings-to-price (EP) ratio. This choice is motivated by evidence from Liu et al. (2019) [27] indicating that in the Chinese market, value effects are better captured by EP than by the book-to-market (BM) ratio commonly used in the U.S.

Following established methodologies, the EP factor is constructed as a long-short portfolio. Quarterly net profit data and monthly/daily return data are sourced from the CSMAR database.

The ME factor exhibits a significantly positive risk-return relationship in both the intercept-included and intercept-free univariate regressions, with coefficients of 4.304 and 5.030, respectively. The EP factor also shows a significantly positive coefficient of 0.149, significant at the 10% level. Although the magnitude is modest, this result confirms a statistically significant risk-return relationship for the value factor proxied by EP.

### 4.3.4. Joint tests.

Following the methodologies of Guo et al. (2009) [17] and Barroso et al. (2025) [25], we conduct joint tests to more rigorously examine the risk-return relationships of non-market factors and their market-linked risks, aiming to further evaluate the consistency with the two-factor ICAPM framework. Using pooled OLS, we estimate the following model:

$$F_{t+1} = \theta_1 RCM_t + \theta_2 RV_{F,t} + \varepsilon_{F,t+1} \tag{19}$$

$$MKT_{t+1} = \theta_1 RV_{M,t} + \theta_2 RCM_t + \varepsilon_{M,t+1} \tag{20}$$

Variable definitions remain consistent with earlier sections. The coefficients $\theta_1$ and $\theta_2$ are constrained to be equal across equations. Here, $\theta_1$ represents the unified price of market volatility risk and factor-market covariance risk, while $\theta_2$ denotes the unified price of factor-specific volatility risk and market-factor covariance risk.

Estimations are performed separately for the size (SMB) and value (HML) factors. Key findings include: The coefficients for SMB and HML $\theta_2$ are 4.008 and 4.538, respectively, both highly significant, indicating that factor-specific volatility risk commands significant positive compensation. Market covariance risk with SMB (HML) also earns significant positive compensation, supporting ICAPM predictions that bearing systematic risk is rewarded with higher expected returns.

In contrast, The coefficients for market volatility risk ($\theta_1$), at 0.144 and 0.400, are statistically insignificant and economically small, suggesting that neither the covariance risk of SMB (HML) with the market nor market volatility itself is priced in predicting next-month returns. This results aligns with the commonly documented "ambiguous market risk-return relationship" in the literature.

In summary, while both the standalone volatility and market covariance risks of the size (SMB) and value (HML) factors are significantly priced and command a unified risk premium, market volatility risk remains statistically undetectable in the time series, indicating a partial alignment with ICAPM theory.

## 4.4. Multivariate Regression

Previous analyses reveal a robust and significant positive risk-return relationship for both the size (SMB) and value (HML) factors. To investigate whether the interaction between these two factors contains predictive information, we extend the baseline model by including the realized covariance between SMB and HML:

$$\mathrm{RC_t} = \sum_{\mathrm{n=0}}^{\mathrm{N}} \mathrm{SMB}_{\mathrm{d_{t-n}}} \cdot \mathrm{HML}_{\mathrm{d_{t-n}}} \tag{21}$$

The bivariate regression results display as follows:

$$SMB_{t+1} = 2.49(0.82)RC_t + 4.86(3.47)RV_t, R^2 = 4.53\%$$

$$HML_{t+1} = 7.81(2.32)RC_t + 5.04(2.36)RV_t, R^2 = 7.06\%$$

The bivariate regression results indicate that: returns of the SMB factor are driven solely byits own volatility risk, with no significant influence from its covariance with the value factor. In contrast, returns of the HML factor are influenced not only by its own volatility risk but also by its synergistic covariation with the SMB factor.

We further introduce the realized covariance with the market factor, which captures exposure to market risk and represents a core hedging motive in the ICAPM framework:

$$SMB_{t+1} = -2.35(-0.91)RCM_t + 2.75(0.90)RC_t + 5.37(3.32)RV_t, R^2 = 4.82\%$$

$$HML_{t+1} = -1.49(-0.73)RCM_t + 8.33(2.28)RC_t + 5.23(2.39)RV_t, R^2 = 7.26\%$$

The results further corroborate earlier conclusions: for the SMB factor, only its own volatility risk remains a significant and stable predictor. Neither its covariance with the market nor its covariance with HML provides incremental predictive power. For the HML factor, its own volatility risk remains significant. Additionally, the covariance between SMB and HML emerges as a significant positive predictor. This suggests that higher comovement between the value and size factors predicts higher future returns for HML. The covariance of HML with the market shows no significant predictive ability.

A potential interpretation is that when value and size effects occur simultaneously, they may represent a combined exposure to business cycle-sensitive risks. Investors may require additional compensation for this compounded risk, leading to higher expected returns during periods of high covariation between the two factors.

## 5. CONCLUSION

This study contributes to the literature on the intertemporal risk–return relationship by shifting the focus from the aggregate market to individual equity factors. Drawing on conditional asset pricing theory, we examine whether the theoretical predictions of the Arbitrage Pricing Theory (Ross, 1976) [10] and the Intertemporal CAPM (Merton, 1973) [11] hold for widely studied factor returns. Using both univariate and multivariate conditional models within the frameworks of Carhart (1997) [12] and Fama and French (2015, 2016) [13, 14], we test for a positive link between factor return and their conditional variances—proxied by realized variance computed from daily returns.

Our results reveal a significant and economically meaningful positive risk–return tradeoff for the size (SMB) and value (HML) factors. These findings are robust to the inclusion of covariance with the market, alternative factor definitions, different business cycle regimes, and bootstrap-based inference. The consistency of these results with the conditional APT and ICAPM suggests that SMB and HML returns compensate investors for factor-specific risk rather than market-linked exposure. In contrast, we find no statistically reliable evidence of a similar tradeoff for other factors, such as momentum (UMD), implying that their returns may stem from sources other than systematic risk compensation—such as behavioral biases or market frictions.

This study refines the theoretical understanding of dynamic asset pricing by empirically validating that the intertemporal risk-return relationship, as postulated by Merton's ICAPM and the APT, operates more distinctly at the factor level—particularly for size and value—than for the aggregate market.The findings offer insights for investors, highlighting that conditioning strategies on the conditional variance of size and value factors can enhance risk-managed returns and improve timing in factor-based allocation.

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