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Optimal Market Encroachment and Pricing Strategy to Form a Global Supply Chain with Strategic Consumers

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ABSTRACT

This paper constructs theoretical game models to explore the joint effects of tax rate disparity and consumers' strategic purchasing behavior on brand's foreign market encroachment strategy and the downstream enterprises' pricing strategy, as well as supply chain members' profit. Results show that (1) Under selling subsidiary encroachment, uniform pricing strategy is only adopted when strategic consumers' patience level is relatively high and the tax rate disparity is sufficiently low. Under retail encroachment, the retailer prefers dynamic pricing strategy when strategic consumers' patience level is relatively low, and uniform pricing strategy otherwise. (2) When dynamic pricing strategy is chosen, a larger tax rate disparity can benefit the selling subsidiary and higher strategic consumers' patience level may benefit both the brand and the selling subsidiary. (3) When strategic consumers' patience level is relatively low, with sufficiently large tax rate disparity, the brand would always choose retail encroachment strategy. With relatively small tax rate disparity, the brand may choose the selling subsidiary encroachment if the establishing cost is not so high, and retail encroachment otherwise. However, when strategic consumers' patience level is relatively high, the brand may also choose selling subsidiary encroachment strategy even if the tax rate disparity is not that high, regardless of what the establishing cost is. This is probably due to retailer's dynamic pricing strategy can bring more demand than the selling subsidiary's uniform pricing strategy does.

KEYWORDS

Global supply chain design; Market encroachment strategy; Pricing strategy; Strategic consumers

1. INTRODUCTION

With the fast development of globalization, enterprises have more incentives to expand their businesses to foreign markets to become multinational firms. One of the most significant issues after targeting the foreign market is determining the encroachment strategy to enter the market. One possible encroachment strategy is to establish a selling subsidiary in the foreign market. The enterprise can benefit from this selling subsidiary encroachment strategy by directly collecting revenues from the foreign market, yet at the expense of establishing cost. If the enterprise plans to save the establishing cost, he can also choose to sell the product through an incumbent foreign retailer, which is called retail encroachment. However, the enterprise has to share profits with the supply chain partner in this decentralized supply chain structure. Furthermore, there is usually tax rate disparity between different countries, which would highly influence multinational firms' plan of allocating profits in the supply chain. As a result, existing studies pay much attention to the effects of tax/tariffs on global supply chain design, especially multinational firms' encroachment strategy. Aside from tax/tariffs, the characteristics of foreign market also play important roles in global supply chain design, such as consumer behavior. Among others, consumers' strategic purchasing behavior is more likely to impose significant impacts, as this intertemporal behavior can largely alter the downstream enterprises' pricing strategy, hence extend its effects on multinational firms' encroachment strategy.

However, effects of consumer behavior in global supply chain management are largely ignored in existing literature.

In view of this background, we are motivated to explore the following research questions: (1) What is the optimal pricing strategy of downstream enterprises in each encroachment strategy of the brand, as well as corresponding equilibrium prices and profit? (2) What is the optimal encroachment strategy for the brand under different circumstances? (3) What are the impacts of tax rate disparity and strategic consumers' patience level on supply chain members' decisions and profit?

Our main findings are as follows: (1) Under selling subsidiary encroachment, uniform pricing strategy is only adopted when strategic consumers' patience level is relatively high and the tax rate disparity is sufficiently low. Otherwise, dynamic pricing strategy would always be optimal for the selling subsidiary. Under retail encroachment, the retailer prefers dynamic pricing strategy when strategic consumers' patience level is relatively low, and uniform pricing strategy otherwise. Note that the impact of tax rate disparity is absent in this circumstance. (2) Counterintuitively, when dynamic pricing strategy is chosen, a larger tax rate disparity can benefit the selling subsidiary, and higher strategic consumers' patience level may benefit both the brand and the selling subsidiary. (3) When strategic consumers' patience level is relatively low, with sufficiently large tax rate disparity, the brand would always choose retail encroachment strategy. With relatively small tax rate disparity, the brand may choose the selling subsidiary encroachment if the establishing cost is not so high, and retail encroachment otherwise. However, when strategic consumers' patience level is relatively high (e.g. $\delta > \frac{1}{2}$), the brand may also choose selling subsidiary encroachment strategy even if the tax rate disparity is not that high, regardless what the establishing cost is. This is probably due to retailer's dynamic pricing strategy can bring more demand than the selling subsidiary's uniform pricing strategy does.

The rest of the paper is organized as follows: Section 2 reviews relevant studies to highlight our theoretical contribution; Section 3 describes the problem setting and defines related notations; Section 4 discusses the scenario under selling subsidiary encroachment; Section 5 then explores the scenario under retail encroachment; Section 6 compares the two scenarios to explore the brand's optimal encroachment strategy; Section 7 concludes the paper.

2. LITERATURE REVIEW

Our work is most closely related to two streams of literature, namely global supply chain management, and operations management with strategic consumers.

With the accelerating development of globalization, global supply chain management has become a hot topic in both industry and academia. Existing literature mainly concentrate on effects of tax/tariffs on multinational firm's pricing strategy (Wu et al., 2018) [1], manufacturing location strategies (Xu et al., 2024) [2], outsourcing and sourcing strategies (Deng and Xu, 2023; Lai et al., 2021; Niu et al., 2019; Xu et al., 2018; Li et al., 2007) [3-7], production mode and production shifting decisions (Lin et al, 2023; Yang et al., 2021; Niu et al., 2020) [8-10], global facility network design (Dong and Kouvelis, 2020; Dong et al., 2010) [11, 12], supply chain design (Deng et al., 2023; Jiang et al., 2023; Hsu and Zhu, 2011) [13-15], and co-opetition strategy (Hsu et al, 2019) [16]. Existing studies do not consider impacts of consumer behavior on global supply chain management. In contrast, this paper explores market encroachment strategy in global supply chain design considering the joint impacts of consumers' strategic purchasing behavior and tax rate disparity.

When determining encroachment strategy to form a global supply chain, market characteristics such as consumer behavior play important roles, aside from tax/tariffs factors. Among others, consumers' strategic purchasing behavior is one of the most significant type. In the past decade, operations management with strategic consumers has always been a hot topic in both industry and academia.

Many scholars concentrate on the effects consumers' strategic purchasing behavior on operations decisions and strategy choice, such as product variety management (Parlaktürk, 2012) [17], group-buying mechanism design (Ke et al. 2017) [18], and capacity decision (Liu & Van Ryzin, 2008) [19] , more researches focus on product pricing, such as pricing of seasonal products (Aviv & Pazgal, 2008) [20], pricing with multi-unit demand (Elmaghraby et al., 2008) [21], pricing in a competitive environment (Levin et al., 2009; Liu & Zhang, 2013) [22, 23], pricing with speculators (Su, 2010) [24], and price commitment/discount strategy (Cachon & Feldman, 2015) [25] etc. However, the above literature study decisions of the vertically integrated enterprises, and do not consider the effects of strategic consumers in a decentralized supply chain setting. In contrast, He et al. (2023) study channel strategy facing strategic consumers [26], Yang et al., (2023) study supply chain co-opetition strategy in the presence of strategic consumers [27], Su & Zhang (2008) examine the impacts of strategic consumers on supply chain coordination [28]. Differently, this paper focuses on the impacts of strategic consumers on encroachment and pricing strategies in global supply chain design.

3. PROBLEM DESCRIPTION

Consider a brand (denoted as B) that is planning to develop its business beyond the local region and encroach a foreign market to sell its product. The brand has two strategies to accomplish the encroachment: (1) To sell through a foreign retailer (denoted as R), which is called retail encroachment; and (2) To establish a selling subsidiary (denoted as S) in the foreign region, which is called selling subsidiary encroachment. The fixed cost of establishing and operating the selling subsidiary is K.

In both encroachment strategies, the brand determines the wholesale price w charged from the foreign retailer/the selling subsidiary, then the foreign retailer/the selling subsidiary determines the retail pricing strategy. If uniform pricing strategy is chosen, the foreign retailer/the selling subsidiary determines the retail price p. If dynamic pricing strategy is chosen, the foreign retailer/the selling subsidiary determines the retail price p_1 in the first period, and then retail price p_2 in the second period. Following from Hsu, Xiao & Xu (2019), this paper assumes the brand has 0 variable cost to produce the product, and the selling subsidiary is a semi-dependent department of the brand, which is granted to make the pricing decision due to having more knowledge of market demand than the headquarter of the brand does. The selling subsidiary maximizes its own profit, and the brand maximizes the total profit consisting of the selling subsidiary's profit and the profit gained in the local region. All the enterprises have to pay tax to the governments of the two regions. The tax rate in the local region is τ, and the gap between the tax rate of the two regions is Δ. Also flowing from Xiao & Xu (2019), it is assumed that the tax rate in the foreign market is higher, i.e. $0 < \Delta < 1 - \tau$.

On the demand side, the foreign market volume is normalized to 1, and the consumers' valuation toward the product v is also normalized and conforms a uniform distribution from 0 to 1, i.e. $v \sim U[0, 1]$. If dynamic pricing is adopted, the consumers discount their utility in the second period comparing with that in the first period, and the discount factor is δ, where $0 < \delta < 1$.

The notations used in this paper are summarized in Table 1.

Table 1. Notations and definitions

Notations	Definition
$\boldsymbol{v}$	Consumers' valuation toward the product, $v \sim U[0,1]$
$\boldsymbol{\delta}$	Consumers' patience level for waiting to make purchase in the second period
$\boldsymbol{K}$	The fixed cost of establishing and operating the selling subsidiary
$\boldsymbol{w}$	The wholesale price charged by the brand from the foreign retailer/the selling subsidiary
$\boldsymbol{p}$	The retail price determined by the retailer/selling subsidiary under uniform pricing strategy
$\boldsymbol{p_i}$	The retail price in the ith period determined by the retailer/selling subsidiary under dynamic pricing strategy, $i = 1,2$
$\boldsymbol{\tau}$	The tax rate levied by the government of the local region
$\boldsymbol{\Delta}$	The gap between the tax rate of the two regions
$\boldsymbol{\Pi_B}$	The profit of the brand
$\boldsymbol{\Pi_S}$	The profit of the selling subsidiary
$\boldsymbol{\Pi_R}$	The profit of the foreign retailer

4. SELLING SUBSIDIARY'S PRICING STRATEGY

When the brand chooses the strategy of selling subsidiary encroachment, the brand establishes a selling subsidiary in the foreign region with fixed cost K to sell its product. If the selling subsidiary chooses dynamic pricing strategy, a consumer with valuation v buys the product in the first period if and only if $v - p_1 \geq \delta(v - p_2)$. In contrast, the consumer buys the product in the second period if and only if $v - p_1 < \delta(v - p_2)$ and $v - p_2 \geq 0$. Hence the demand is $1 - \frac{p_1 - \delta p_2}{1-\delta}$ in the first period and $\frac{p_1 - \delta p_2}{1-\delta} - p_2$ in the second period. Therefore, as a semi-dependent department, the selling subsidiary maximizes its own profit, and faces the problem

$$\mathrm{Max}_{p_1,p_2}\Pi_S = ((p_1 - w)\left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right) + (p_2 - w)(\frac{p_1 - \delta p_2}{1-\delta} - p_2))(1 - (\tau + \Delta))$$

$$\text{s.t. } 0 < w < p_2 < p_1 < 1$$

The brand maximizes the total profit including the selling subsidiary's profit as well as the head quarter's obtained in the local region, facing the problem

$$\mathrm{Max}_w\Pi_B = w(1 - p_2)(1 - \tau) + \left((p_1 - w)\left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right) + (p_2 - w)\left(\frac{p_1 - \delta p_2}{1-\delta} - p_2\right)\right)\big(1 - (\tau + \Delta)\big) - K$$

$$\text{s.t. } 0 < w < p_2 < p_1 < 1$$

This paper uses standard backward induction to analyze this Stackelberg game. Proposition 1 is utilized to characterize the equilibrium outcome of wholesale and retail prices as well as the brand's and the selling subsidiary's profit in this circumstance.

Proposition 1 When the brand adopts selling subsidiary encroachment strategy and the selling subsidiary adopts dynamic pricing strategy,

(1) The selling subsidiary's best responses are $p_1 = \frac{w+2(1-\delta)}{3-2\delta}$ and $p_2 = \frac{p_1+w}{2} = \frac{(2-\delta)w+(1-\delta)}{3-2\delta}$.

(2) The equilibrium wholesale and retail prices are $w^* = \frac{2(1-\delta)\Delta+\delta(1-\tau)}{2((1-\delta)\Delta+(1-\tau))}$, $p_1^* = \frac{(1-\tau)(4-3\delta)+2(1-\delta)(3-2\delta)\Delta}{2(1-\tau+\Delta(1-\delta))(3-2\delta)}$, $p_2^* = \frac{(1-\tau)(2-\delta^2)+2(1-\delta)(3-2\delta)\Delta}{2(1-\tau+\Delta(1-\delta))(3-2\delta)}$, respectively.

(3) The equilibrium profit of the brand and the selling subsidiary are $\Pi_B^* = \frac{(2-\delta)^2(1-\tau)^2-4K(1-\tau+\Delta(1-\delta))(3-2\delta)}{4(1-\tau+\Delta(1-\delta))(3-2\delta)}$ and $\Pi_S^* = \frac{(2-\delta)^2(1-\tau)^2(1-\delta)(1-(\tau+\Delta))}{4(1-\tau+\Delta(1-\delta))^2(3-2\delta)}$, respectively.

Proof: All the proofs in this paper are delegated to the Appendix.

Proposition 1 shows that the tax rate disparity and consumers' patience level both exert different impacts on equilibrium prices and profit: a larger tax rate disparity induces higher equilibrium wholesale price and retail prices, but lower brand's profit. However, the selling subsidiary's profit first decreases then increases in tax rate disparity, which exhibits a non-monotonic characteristic. Meanwhile, a higher consumers' patience level induces higher equilibrium wholesale price yet lower retail price in the first period. However, retail price in the second period first increases then decreases in consumers' patience level. The brand's and selling subsidiary's profit both decrease and then increases in consumers' patience level. This is quite different from conventional wisdom that consumers' strategic purchasing behavior always undermines enterprises' profit.

When the selling subsidiary adopts uniform pricing strategy, the selling subsidiary maximizes his own profit by setting the retail price p given the brand's wholesale price w, hence faces the problem

$$\text{Max}_p \Pi_S = (p-w)(1-p)(1-(\tau+\Delta))$$

$$\text{s.t. } 0 < w < p < 1$$

Correspondingly, the brand faces the problem

$$\text{Max}_w \Pi_B = w(1-p)(1-\tau) + (p-w)(1-p)\big(1-(\tau+\Delta)\big) - K$$

$$\text{s.t. } 0 < w < p < 1$$

The equilibrium outcome of this game is characterized in Proposition 2.

Proposition 2 When the brand adopts selling subsidiary encroachment strategy and the selling subsidiary adopts uniform pricing strategy,

(1) The selling subsidiary's best response is $p = \frac{1+w}{2}$.

(2) The equilibrium wholesale and retail prices are $w^* = \frac{\Delta}{1-\tau+\Delta}$, $p^* = \frac{1-\tau+2\Delta}{2(1-\tau+\Delta)}$, respectively.

(3) The equilibrium profit of the brand and the selling subsidiary are $\Pi_B^* = \frac{(1-\tau)^2-4K(1-\tau+\Delta)}{4(1-\tau+\Delta)}$ and $\Pi_S^* = \frac{(1-\tau)^2(1-(\tau+\Delta))}{4(1-\tau+\Delta)^2}$, respectively.

Proposition 2 implicates that, consumers' patience level does not impact the decision and profit as the selling subsidiary adopts uniform pricing strategy. On the contrary, a larger tax rate disparity pushes up the equilibrium wholesale and retail prices, and undermines the brand's and selling subsidiary's profit.

Based on the above equilibrium outcome analyses on the selling subsidiary's two pricing strategies, we then examine how the selling subsidiary should choose pricing strategy by comparing its profit under these two pricing strategies. Proposition 3 provides the optimal pricing strategy under the brand's selling subsidiary encroachment.

Proposition 3 When the brand adopts selling subsidiary encroachment strategy, the selling subsidiary's optimal pricing strategy facing strategic consumers is given as follows:

(1) When $\delta \leq \delta^*$, the selling subsidiary's optimal pricing strategy is dynamic pricing, where δ^* is the solution of $\delta^3 - 5\delta^2 + 6\delta - 1 = 0$.

(2) When $\delta > \delta^*$, if $\Delta < \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)} - (1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is uniform pricing; if $\Delta \geq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)} - (1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is dynamic pricing.

Proposition 3 suggests that when consumers' patience level is relatively low, the selling subsidiary always adopts dynamic pricing strategy to price skimming the consumers, as consumers' strategic waiting for low price is not so prominent. However, when strategic waiting becomes more and more attractive, the selling subsidiary may have to adopt uniform pricing strategy to induce consumers to buy early at the high price level, especially when the tax rate disparity is low. When the tax rate disparity is high, the impact of consumers' high patience level is offset, hence the selling subsidiary still adopts dynamic pricing strategy.

5. RETAILER'S PRICING STRATEGY

When the brand chooses the strategy of retail encroachment, the brand sells the product through a incumbent retailer in the foreign market. If the selling subsidiary chooses dynamic pricing strategy, a consumer with valuation v satisfying $v - p_1 \geq \delta(v - p_2)$ buys the product in the first period. Also, the consumer buys the product in the second period if and only if $v - p_1 < \delta(v - p_2)$ and $v - p_2 \geq 0$. Hence the demand is $1 - \frac{p_1 - \delta p_2}{1-\delta}$ in the first period and $\frac{p_1 - \delta p_2}{1-\delta} - p_2$ in the second period. Therefore, the retailer faces the problem similar to the selling subsidiary

$$\text{Max}_{p_1, p_2} \Pi_R = ((p_1 - w)\left(1 - \frac{p_1 - \delta p_2}{1 - \delta}\right) + (p_2 - w)(\frac{p_1 - \delta p_2}{1 - \delta} - p_2))(1 - (\tau + \Delta))$$

$$\text{s.t. } 0 < w < p_2 < p_1 < 1$$

However, the brand under retail encroachment does not include the downstream profit any more, and only maximizes the local profit, facing the problem

$$\text{Max}_w \Pi_B = w(1 - p_2)(1 - \tau)$$

$$\text{s.t. } 0 < w < p_2 < p_1 < 1$$

Proposition 4 is utilized to characterize the equilibrium outcome of wholesale and retail prices as well as the brand's and the retailer's profit in this circumstance.

Proposition 4 When the brand adopts retail encroachment strategy and the retailer adopts dynamic pricing strategy,

(1) The retailer's best responses are $p_1 = \frac{w + 2(1-\delta)}{3 - 2\delta}$ and $p_2 = \frac{p_1 + w}{2} = \frac{(2-\delta)w + (1-\delta)}{3-2\delta}$.

(2) The equilibrium wholesale and retail prices are $w^* = \frac{1}{2}$, $p_1^* = \frac{5-4\delta}{2(3-2\delta)}$, $p_2^* = \frac{4-3\delta}{2(3-2\delta)}$, respectively.

(3) The equilibrium profit of the brand and the selling subsidiary are $\Pi_B^* = \frac{(2-\delta)(1-\tau)}{4(3-2\delta)}$ and $\Pi_R^* = \frac{(1-\delta)(1-(\tau+\Delta))}{4(3-2\delta)}$, respectively.

Proposition 4 shows that the tax rate only impacts the retailer's profit and has no impact on the equilibrium prices and the brand's profit. Specifically, a larger tax rate disparity undermines the retailer's profit. On the contrary, supply chain members' decision and profit all bear impacts from strategic consumers' patience level, except the wholesale price. In particular, the retail prices in both periods, as well as the retailer's profit, all decreases in consumers' patience level. However, the brand benefits from a larger consumers' patience level.

When the retailer adopts uniform pricing strategy, the retailer maximizes his own profit by setting the retail price p given the brand's wholesale price w, hence faces the problem

$$\text{Max}_p \Pi_R = (p-w)(1-p)(1-(\tau+\Delta))$$

$$\text{s.t. } 0 < w < p < 1$$

Correspondingly, the brand faces the problem

$$\text{Max}_w \Pi_B = w(1-p)(1-\tau)$$

$$\text{s.t. } 0 < w < p < 1$$

The equilibrium outcome of this game is characterized in Proposition 5.

Proposition 5 When the brand adopts retail encroachment strategy and the selling subsidiary adopts uniform pricing strategy,

(1) The retailer's best response is $p = \frac{1+w}{2}$.

(2) The equilibrium wholesale and retail prices are $w^* = \frac{1}{2}$, $p^* = \frac{1-\tau+2\Delta}{2(1-\tau+\Delta)}$, respectively.

(3) The equilibrium profit of the brand and the selling subsidiary are $\Pi_B^* = \frac{1-\tau}{8}$ and $\Pi_R^* = \frac{1-(\tau+\Delta)}{16}$, respectively.

Proposition 5 implicates that, the equilibrium prices and profit bear less impacts from the tax rate disparity and consumers' strategic purchasing behavior, due to the complete decentralized supply chain structure and retailer's simple pricing strategy. As a result, only the retailer's profit decreases in the tax rate disparity. Other impacts from tax rate disparity and strategic consumer behavior are absent.

Based on the above equilibrium outcome analyses on the retailer's two pricing strategies, we then examine how the retailer should choose pricing strategy by comparing its profit under these two pricing strategies. Proposition 6 provides the optimal pricing strategy under the brand's retail encroachment.

Proposition 6 When the brand adopts retail encroachment strategy, the retailer's optimal pricing strategy facing strategic consumers is given as follows:

(1) When $\delta \leq \frac{1}{2}$, the retailer's optimal pricing strategy is dynamic pricing.

(2) When $\delta > \frac{1}{2}$, the retailer's optimal pricing strategy is uniform pricing.

Proposition 6 suggests that when the brand adopts retail encroachment strategy to encroach a foreign market with low strategic consumer patience level, the retailer would adopt dynamic pricing strategy. Otherwise, the retailer would adopt uniform pricing strategy. This is in consistent with the results of extant literature.

6. THE BRAND'S ENCROACHMENT STRATEGY

Integrating the results of Sections 4 and 5, it is possible to examine the brand's optimal encroachment strategy by comparing the brand's profit under various scenarios. Proposition 7 gives the equilibrium outcome in this scenario.

Proposition 7 The brand's optimal encroachment strategy under various conditions is given as follows:

(1) $0 < \delta \leq \delta^*$,

When $\Delta < \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{(1-\delta)(4(3-2\delta)K+(2-\delta)(1-\tau))}$, if $K < \frac{(1-\delta)(2-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment; if $K \geq \frac{(1-\delta)(2-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment.

When $\Delta \geq \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{(1-\delta)(4(3-2\delta)K+(2-\delta)(1-\tau))}$, the brand's optimal encroachment strategy is retail encroachment.

(2) $\delta^* < \delta \leq \frac{1}{2}$,

When $\Delta < \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, if $K \geq \frac{(1-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment; if $K < \frac{(1-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment.

When $\frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)} \leq \Delta < \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{4(1-\delta)(3-2\delta)K+((1-\delta)(2-\delta)(1-\tau)}$, if $K \geq \frac{(2-\delta)(1-\tau)(8(1-\delta)-4\sqrt{(1-\delta)(3-2\delta)})}{16\delta(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment; if $K < \frac{(2-\delta)(1-\tau)(8(1-\delta)-4\sqrt{(1-\delta)(3-2\delta)})}{16\delta(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment

When $\Delta \geq \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{4(1-\delta)(3-2\delta)K+((1-\delta)(2-\delta)(1-\tau)}$, the brand's optimal encroachment strategy is retail encroachment.

(3) $\frac{1}{2} < \delta < 1$,

When $\Delta < \frac{(1-\tau)(1-\tau-8K)}{8K+(1-\tau)}$, if $K < \frac{1-\tau}{8}$, the brand's optimal encroachment strategy is selling subsidiary encroachment; if $K \geq \frac{1-\tau}{8}$, the brand's optimal encroachment strategy is retail encroachment.

When $\frac{(1-\tau)(1-\tau-8K)}{8K+(1-\tau)} \leq \Delta < \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the brand's optimal encroachment strategy is retail encroachment.

When $\frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)} \leq \Delta < \frac{(1-\tau)((5-6\delta+2\delta^2)(1-\tau)-8(3-2\delta)K)}{(1-\delta)(3-2\delta)(8K+1-\tau)}$, if $K < \frac{(1-\tau)(2\delta^2-\frac{11}{2}\delta+4-(2-\delta)\sqrt{(1-\delta)(3-2\delta)})}{4\delta(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment; if $K \geq \frac{(1-\tau)(2\delta^2-\frac{11}{2}\delta+4-(2-\delta)\sqrt{(1-\delta)(3-2\delta)})}{4\delta(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment.

When $\Delta \geq \frac{(1-\tau)((5-6\delta+2\delta^2)(1-\tau)-8(3-2\delta)K)}{(1-\delta)(3-2\delta)(8K+1-\tau)}$, the brand's optimal encroachment strategy is retail encroachment.

Proposition 7 shows that when the tax rate disparity is sufficiently large, retail encroachment strategy would always be optimal for the brand. Otherwise, the establishing cost of setting a selling subsidiary may count. That is to say, the brand may choose to set a selling subsidiary only when the establishing cost is not so high. However, higher strategic consumers' patience level can complicate the situation. Specifically, when $\delta > \frac{1}{2}$, the brand may also choose retail encroachment strategy even if the tax rate disparity is not that high, regardless what the establishing cost is. This is probably due to retailer's dynamic pricing strategy can bring more demand than the selling subsidiary's uniform pricing strategy does.

7. CONCLUSION

With the fast development of globalization, enterprises have more incentives to expand their businesses to foreign markets to become multinational firms. When exploring the foreign market encroachment strategy and downstreams' pricing strategy, the effects of tax rate disparity is well discussed. However, the effects of market characteristics such as consumers' strategic purchasing behavior is largely ignored. Therefore, this paper utilizes mathematical models of game theory to study the joint effects of tax rate disparity and consumers' strategic purchasing behavior when forming a global supply chain. The results show that: when the brand adopts selling subsidiary encroachment strategy, if the selling subsidiary adopts dynamic pricing strategy, a larger tax rate disparity induces higher equilibrium wholesale price and retail prices, but lower brand's profit. However, the selling subsidiary's profit first decreases then increases in tax rate disparity. Meanwhile, a higher consumers' patience level induces higher equilibrium wholesale price yet lower retail price in the first period. However, retail price in the second period first increases then decreases in consumers' patience level. The brand's and selling subsidiary's profit both decrease and then increases in consumers' patience level. If the selling subsidiary adopts uniform pricing strategy, consumers' patience level does not impact the decision and profit as the selling subsidiary adopts uniform pricing strategy. On the contrary, a larger tax rate disparity pushes up the equilibrium wholesale and retail prices, and undermines the brand's and selling subsidiary's profit. Comparing the two cases with different pricing strategies of the selling subsidiary, when consumers' patience level is relatively low, the selling subsidiary always adopts dynamic pricing strategy to price skimming the consumers. However, when strategic waiting becomes more and more attractive, the selling subsidiary may have to adopt uniform pricing strategy to induce consumers to buy early at the high price level, especially when the tax rate disparity is low. When the tax rate disparity is high, the impact of consumers' high patience level is offset, hence the selling subsidiary still adopts dynamic pricing strategy.

When the brand adopts retail encroachment strategy, if the retailer adopts dynamic pricing strategy, the tax rate only impacts the retailer's profit and has no impact on the equilibrium prices and the brand's profit. Specifically, a larger tax rate disparity undermines the retailer's profit. On the contrary, supply chain members' decision and profit all bear impacts from strategic consumers' patience level, except the wholesale price. In particular, the retail prices in both periods, as well as the retailer's

profit, all decreases in consumers' patience level. However, the brand benefits from a larger consumers' patience level. If the retailer adopts uniform pricing strategy, the equilibrium prices and profit bear less impacts from the tax rate disparity and consumers' strategic purchasing behavior, due to the complete decentralized supply chain structure and retailer's simple pricing strategy. As a result, only the retailer's profit decreases in the tax rate disparity. Other impacts from tax rate disparity and strategic consumer behavior are absent. Comparing the retailer's two pricing strategies, the retailer would adopt dynamic pricing strategy with low strategic consumer patience level. Otherwise, the retailer would adopt uniform pricing strategy.

When the tax rate disparity is sufficiently large, retail encroachment strategy would always be optimal for the brand. Otherwise, the brand may choose to set a selling subsidiary when the establishing cost is not so high. However, when strategic consumers' patience level is sufficiently high, the brand may choose retail encroachment strategy even if the tax rate disparity is at a medium level, regardless what the establishing cost is. This is probably due to retailer's dynamic pricing strategy can bring more demand than the selling subsidiary's uniform pricing strategy does.

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APPENDIX

Proof of Proposition 1:

Following from the first order condition of $\Pi_{S2} = (p_2 - w)(\frac{p_1 - \delta p_2}{1-\delta} - p_2)(1 - (\tau + \Delta))$ w.r.t. p_2, it can be obtained that the optimal retail price in the second period is $p_2^* = \frac{p_1 + w}{2}$, given w and p_1. Substituting it into $\Pi_S = ((p_1 - w)\left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right) + (p_2 - w)(\frac{p_1 - \delta p_2}{1-\delta} - p_2))(1 - (\tau + \Delta))$ and analyzing its first order condition w.r.t. p_1, it can be found that the optimal retail price in the first period is $p_1^* = \frac{w + 2(1-\delta)}{3-2\delta}$, given w. Further, $p_2^* = \frac{(2-\delta)w + (1-\delta)}{3-2\delta}$.

With similar logic, substituting $p_1^* = \frac{w + 2(1-\delta)}{3-2\delta}$ and $p_2^* = \frac{(2-\delta)w + (1-\delta)}{3-2\delta}$ into $\Pi_B = w(1 - p_2)(1 - \tau) + \left((p_1 - w)\left(1 - \frac{p_1 - \delta p_2}{1-\delta}\right) + (p_2 - w)\left(\frac{p_1 - \delta p_2}{1-\delta} - p_2\right)\right)\left(1 - (\tau + \Delta)\right) - K$ and analyzing its first order condition w.r.t. w, it can be obtained that $w^* = \frac{2(1-\delta)\Delta + \delta(1-\tau)}{2((1-\delta)\Delta + (1-\tau))}$, hence $p_1^* = \frac{(1-\tau)(4-3\delta) + 2(1-\delta)(3-2\delta)\Delta}{2(1-\tau+\Delta(1-\delta))(3-2\delta)}$, $p_2^* = \frac{(1-\tau)(2-\delta^2) + 2(1-\delta)(3-2\delta)\Delta}{2(1-\tau+\Delta(1-\delta))(3-2\delta)}$. Substituting the equilibrium prices into the brand's and selling subsidiary's profit function, then $\Pi_B^* = \frac{(2-\delta)^2(1-\tau)^2 - 4K(1-\tau+\Delta(1-\delta))(3-2\delta)}{4(1-\tau+\Delta(1-\delta))(3-2\delta)}$, $\Pi_S^* = \frac{(2-\delta)^2(1-\tau)^2(1-\delta)(1-(\tau+\Delta))}{4(1-\tau+\Delta(1-\delta))^2(3-2\delta)}$.

Proof of Proposition 2:

Following from the first order condition of $\Pi_S = (p - w)(1 - p)(1 - (\tau + \Delta))$ w.r.t. p, it can be obtained that the optimal retail price is $p^* = \frac{1+w}{2}$, given w. Substituting it into $\Pi_B = w(1 - p)(1 - \tau) + (p - w)(1 - p)\left(1 - (\tau + \Delta)\right) - K$ and analyzing its first order condition w.r.t.

w, it can be found that the optimal wholesale price is $w^* = \frac{\Delta}{1-\tau+\Delta}$, hence $p^* = \frac{1-\tau+2\Delta}{2(1-\tau+\Delta)}$. Substituting the equilibrium prices into the brand's and selling subsidiary's profit function, then $\Pi_B^* = \frac{(1-\tau)^2-4K(1-\tau+\Delta)}{4(1-\tau+\Delta)}$, $\Pi_S^* = \frac{(1-\tau)^2(1-(\tau+\Delta))}{4(1-\tau+\Delta)^2}$.

Proof of Proposition 3:

Check the selling subsidiary's profit gap $\frac{(1-\tau)^2(1-(\tau+\Delta))}{4(1-\tau+\Delta)^2} - \frac{(2-\delta)^2(1-\tau)^2(1-\delta)(1-(\tau+\Delta))}{4(1-\tau+\Delta(1-\delta))^2(3-2\delta)}$, the following results can be obtained:

When $\delta \leq \delta^*$, where δ^* is the solution of $\delta^3 - 5\delta^2 + 6\delta - 1 = 0$, the selling subsidiary's optimal pricing strategy is dynamic pricing.

When $\delta > \delta^*$, if $\Delta < \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is uniform pricing; if $\Delta \geq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is dynamic pricing.

Proof of Proposition 4:

Following from the first order condition of $\Pi_{R2} = (p_2 - w)(\frac{p_1-\delta p_2}{1-\delta} - p_2)(1-(\tau+\Delta))$ w.r.t. p_2, it can be obtained that the optimal retail price in the second period is $p_2^* = \frac{p_1+w}{2}$, given w and p_1. Substituting it into $\Pi_R = ((p_1 - w)\left(1 - \frac{p_1-\delta p_2}{1-\delta}\right) + (p_2 - w)(\frac{p_1-\delta p_2}{1-\delta} - p_2))(1-(\tau+\Delta))$ and analyzing its first order condition w.r.t. p_1, it can be found that the optimal retail price in the first period is $p_1^* = \frac{w+2(1-\delta)}{3-2\delta}$, given w. Further, $p_2^* = \frac{(2-\delta)w+(1-\delta)}{3-2\delta}$.

With similar logic, substituting $p_1^* = \frac{w+2(1-\delta)}{3-2\delta}$ and $p_2^* = \frac{(2-\delta)w+(1-\delta)}{3-2\delta}$ into $\Pi_B = w(1-p_2)(1-\tau)$ and analyzing its first order condition w.r.t. w, it can be obtained that $w^* = \frac{1}{2}$, hence $p_1^* = \frac{5-4\delta}{2(3-2\delta)}$, $p_2^* = \frac{4-3\delta}{2(3-2\delta)}$. Substituting the equilibrium prices into the brand's and retailer's profit function, then $\Pi_B^* = \frac{(2-\delta)(1-\tau)}{4(3-2\delta)}$, $\Pi_R^* = \frac{(1-\delta)(1-(\tau+\Delta))}{4(3-2\delta)}$.

Proof of Proposition 5:

Following from the first order condition of $\Pi_R = (p-w)(1-p)(1-(\tau+\Delta))$ w.r.t. p, it can be obtained that the optimal retail price is $p^* = \frac{1+w}{2}$, given w. Substituting it into $\Pi_B = w(1-p)(1-\tau)$ and analyzing its first order condition w.r.t. w, it can be found that the optimal wholesale price is $w^* = \frac{1}{2}$, hence $p^* = \frac{3}{4}$. Substituting the equilibrium prices into the brand's and selling subsidiary's profit function, then $\Pi_B^* = \frac{1-\tau}{8}$, $\Pi_R^* = \frac{1-(\tau+\Delta)}{16}$.

Proof of Proposition 6:

Check the retailer's profit gap $\frac{1-(\tau+\Delta)}{16} - \frac{(1-\delta)(1-(\tau+\Delta))}{4(3-2\delta)}$, the following results can be obtained:

When $\delta \leq \frac{1}{2}$, the retailer's optimal pricing strategy is dynamic pricing.

When $\delta > \frac{1}{2}$, the retailer's optimal pricing strategy is uniform pricing.

Proof of Proposition 7:

Integrate the above results to check the brand's optimal encroachment strategy:

(1) $0 < \delta \leq \delta^*$

When the brand adopts selling subsidiary encroachment, the selling subsidiary adopts dynamic pricing strategy; when the brand adopts retail encroachment, the retailer adopts dynamic pricing strategy. Therefore, check the brand's profit gap between these two cases

$$\frac{(2-\delta)^2(1-\tau)^2 - 4K(1-\tau+\Delta(1-\delta))(3-2\delta)}{4(1-\tau+\Delta(1-\delta))(3-2\delta)} - \frac{(2-\delta)(1-\tau)}{4(3-2\delta)}$$

When $K \geq \frac{(1-\delta)(2-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment.

When $K < \frac{(1-\delta)(2-\delta)(1-\tau)}{4(3-2\delta)}$, if $\Delta < \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{(1-\delta)(4(3-2\delta)K+(2-\delta)(1-\tau))}$, the brand's optimal encroachment strategy is selling subsidiary encroachment; if $\Delta \geq \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{(1-\delta)(4(3-2\delta)K+(2-\delta)(1-\tau))}$, the brand's optimal encroachment strategy is retail encroachment.

(2) $\delta^* < \delta \leq \frac{1}{2}$

When the brand adopts selling subsidiary encroachment, if $\Delta < \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is uniform pricing; if $\Delta \geq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is dynamic pricing.
When the brand adopts retail encroachment, the retailer adopts dynamic pricing strategy. Therefore, check the brand's profit gap between these two cases under different scenarios.

When $\Delta \leq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, check $\frac{(1-\tau)^2-4K(1-\tau+\Delta)}{4(1-\tau+\Delta)} - \frac{(2-\delta)(1-\tau)}{4(3-2\delta)}$.

If $K \geq \frac{(1-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment.

If $K < \frac{(1-\delta)(1-\tau)}{4(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment.

When $\Delta > \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, check $\frac{(1-\tau)^2-4K(1-\tau+\Delta)}{4(1-\tau+\Delta)} - \frac{(2-\delta)(1-\tau)}{4(3-2\delta)}$.

When $\frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)} < \Delta < \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{4(1-\delta)(3-2\delta)K+((1-\delta)(2-\delta)(1-\tau)}$, if $K < \frac{(2-\delta)(1-\tau)(8(1-\delta)-4\sqrt{(1-\delta)(3-2\delta)})}{16\delta(3-2\delta)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment; if $K \geq \frac{(2-\delta)(1-\tau)(8(1-\delta)-4\sqrt{(1-\delta)(3-2\delta)})}{16\delta(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment

When $\Delta \geq \frac{(1-\tau)((1-\delta)(2-\delta)(1-\tau)-4(3-2\delta)K)}{4(1-\delta)(3-2\delta)K+((1-\delta)(2-\delta)(1-\tau)}$, the brand's optimal encroachment strategy is retail encroachment.

(3) $\frac{1}{2} < \delta < 1$

When the brand adopts selling subsidiary encroachment, if $\Delta \leq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is uniform pricing; if $\Delta > \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the selling subsidiary's optimal pricing strategy is dynamic pricing.
When the brand adopts retail encroachment, the retailer adopts uniform pricing strategy.

Therefore, check the brand's profit gap between these two cases under different scenarios.

When $\Delta \leq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, check $\frac{(1-\tau)^2-4K(1-\tau+\Delta)}{4(1-\tau+\Delta)} - \frac{1-\tau}{8}$.

If $K \geq \frac{1-\tau}{8}$, the brand's optimal encroachment strategy is retail encroachment.

If $K < \frac{1-\tau}{8}$ and $\Delta \leq \frac{(1-\tau)(1-\tau-8K)}{8K+(1-\tau)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment.

If $K < \frac{1-\tau}{8}$ and $\frac{(1-\tau)(1-\tau-8K)}{8K+(1-\tau)} < \Delta \leq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the brand's optimal encroachment strategy is retail encroachment.

When $\Delta > \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, check $\frac{(2-\delta)^2(1-\tau)^2-4K(1-\tau+\Delta(1-\delta))(3-2\delta)}{4(1-\tau+\Delta(1-\delta))(3-2\delta)} - \frac{1-\tau}{8}$.

If $K \geq \frac{(1-\tau)(2\delta^2-\frac{11}{2}\delta+4-(2-\delta)\sqrt{(1-\delta)(3-2\delta)})}{4\delta(3-2\delta)}$, the brand's optimal encroachment strategy is retail encroachment.

If $K < \frac{(1-\tau)(2\delta^2-\frac{11}{2}\delta+4-(2-\delta)\sqrt{(1-\delta)(3-2\delta)})}{4\delta(3-2\delta)}$ and $\Delta < \frac{(1-\tau)(\left(5-6\delta+2\delta^2\right)(1-\tau)-8(3-2\delta)K)}{8(1-\delta)(3-2\delta)K+((1-\delta)(3-2\delta)(1-\tau)}$, the brand's optimal encroachment strategy is selling subsidiary encroachment.

If $K < \frac{(1-\tau)(2\delta^2-\frac{11}{2}\delta+4-(2-\delta)\sqrt{(1-\delta)(3-2\delta)})}{4\delta(3-2\delta)}$ and $\frac{(1-\tau)(\left(5-6\delta+2\delta^2\right)(1-\tau)-8(3-2\delta)K)}{8(1-\delta)(3-2\delta)K+((1-\delta)(3-2\delta)(1-\tau)} < \Delta \leq \frac{(\delta(2-\delta)\sqrt{(1-\delta)(3-2\delta)}-(1-\delta)^3)(1-\tau)}{(1-\delta)(1+\delta-\delta^2)}$, the brand's optimal encroachment strategy is retail encroachment.

Integrate all the above results, Proposition 7 can be proved.