

Research on Dual-Channel Supply Chain Decisions in Live-Stream E-Commerce under Different Commission Contracts

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ABSTRACT

This paper examines contract choice and pricing decisions in a dual-channel supply chain where a manufacturer sells products through both online direct sales and influencer live-streaming. We develop two Stackelberg game models—a pure commission contract and a commission-plus-service-fee contract—and derive equilibrium solutions via backward induction. We analyze how streamer influence, channel spillover effects, and service fee intensity shape channel pricing, streamer marketing effort, and supply chain profitability. Results show that stronger streamer influence significantly raises channel prices and streamer marketing effort. The commission-plus-service-fee contract outperforms the pure commission contract in motivating streamer effort and lifting channel prices, yet the service fee has an optimal range: an excessively high service fee reduces total supply chain profit. A positive channel spillover effect enhances the profits of all supply chain members and improves system performance. These findings offer theoretical and practical insights for contract design, pricing, and coordinated operations in live-stream e-commerce supply chains.

KEYWORDS

Live-stream e-commerce; Dual-channel supply chain; Contract design; Streamer influence; Spillover effect

1. INTRODUCTION

Live-stream e-commerce has become a dominant engine of China's online retail growth. According to the 2025 Live-Stream E-Commerce Industry Development White Paper, China's live-stream e-commerce GMV surpassed 5 trillion yuan in 2025, accounting for nearly one-third of national online retail sales, with a user base of 660 million. Many manufacturers now operate a dual-channel structure: official online direct sales complemented by influencer live-stream channels [1]. While this structure leverages streamer influence to drive traffic and sales, it also creates channel competition, pricing conflicts, and misaligned profit allocation. As the core incentive mechanism linking manufacturers and streamers, the commission contract directly determines pricing strategies, marketing effort, and overall supply chain performance, making it a critical issue in live-stream commerce research.

Prior studies on commission contracts mostly focus on pure commission structures. For instance, Cui et al. (2023) [2] analyze a platform-based live-stream supply chain with an exogenous commission rate and examine how live-streaming affects pricing, sales, and efficiency. Sun et al. (2024) [3] model tripartite effort and revenue sharing; Qin et al. (2025) [4] investigate traffic modes and pricing under streamer influence.

Other studies incorporate a fixed slotting fee alongside commission. He et al. (2024) [5] compare pure commission, pure slotting fee, and hybrid contracts in a manufacturer–retailer–streamer supply

chain. Wang et al. (2025) [6] show that hybrid contracts can improve prices, sales, and profits across different live-stream formats.

However, existing research rarely provides a systematic comparison between pure commission and commission-plus-service-fee contracts, especially regarding the joint effects of streamer influence, channel spillovers, and service fees on dual-channel decisions. Few studies offer clear thresholds for contract selection. To fill this gap, we build two Stackelberg models—with the manufacturer as leader and the streamer as follower—and derive equilibrium strategies. We then use numerical analysis to validate our results. This paper aims to clarify the performance differences and applicable conditions of the two contracts, supporting better contract design, channel coordination, and efficiency improvement in live-stream supply chains.

2. MODEL DESCRIPTION AND ASSUMPTIONS

2.1. Model Structure

We consider a dual-channel supply chain consisting of one manufacturer and one influencer streamer. The manufacturer sells through an online direct channel and a live-stream channel. In the live-stream channel, the manufacturer pays the streamer a per-transaction commission. The manufacturer acts as the Stackelberg leader; the streamer is the follower.

In the direct sales channel, the manufacturer sells products to consumers at the direct selling price p_1 ; in the live-stream channel, the streamer sells products to consumers at the live-stream price p_2 . The specific structure of the supply chain is shown in Figure 1.

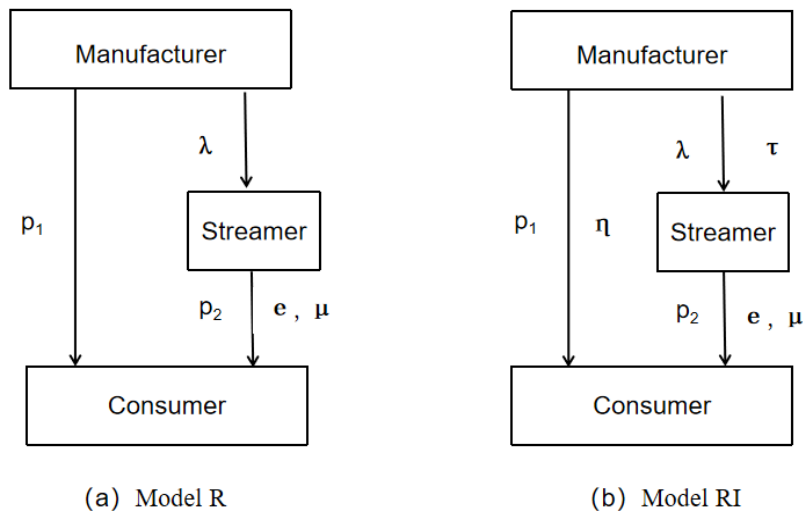


Figure 1. Structure under Different Commission Contract Modes

Model R (Pure Commission): In the direct sales channel, the manufacturer sells to consumers at a direct price of p_1 ; in the livestreaming channel, the manufacturer pays the streamer a commission of λ , while the streamer independently determines the retail price p_2 and the marketing effort e for the product.

Model RI (Commission plus Service Fee): In the live streaming channel, the manufacturer pays the streamer a commission rate of λ for the streamer's commission, and also pays a service fee of $\tau\mu$ to the streamer. Since the manufacturer pays the service fee, the manufacturer can intercept the streamer's live streaming clips for use in the direct sales channel. Due to the streamer's influence denoted as μ , the live streaming channel exerts a positive spillover effect on the direct sales channel, with the spillover coefficient being η , and the spillover represented as $\eta\mu$.

2.2. Basic Assumptions

Production cost and selling cost are normalized to zero (Dong et al., 2025 [7]).

Total market potential is standardized to 1 (Dong et al., 2025 [7]).

The commission rate λ is exogenous (Zhang et al., 2024 [8]).

Slotting fees are negligible and set to zero (Qin et al., 2025) [9].

Consumers have distinct preferences for the live-stream α and direct channels $1-\alpha$ (Li & Chai, 2024 [10]).

Channel competition follows a standard price-competitive demand system, namely $D_2=1-p_2+\theta p_1$, θ represents the cross-price elasticity coefficient, which measures the degree of competition between channels. When $\theta=0$, there is no mutual pricing interference between channels, indicating the lowest level of competition. (Li & Chai, 2024 [10]).

The market equilibrium demand function for the direct sales channel is $D_1=1-\alpha-p_1+\theta p_2$, and that for the live-streaming channel is $D_2=\alpha-p_2+\theta p_1+e+\mu$. (Li & Chai, 2024 [10]; Xiong et al., 2023 [11]).

The model notion and explanations involved in this paper are shown in Table 1.

Table 1. Definitions of Related Parameters

Notation	Explanations
α	the basic market demand for live-stream channel
θ	degree of competition between channels
η	spillover effect coefficient
μ	streamer influence
τ	service fee coefficient
e	streamer marketing effort
p_1	product price in the direct sales channel
p_2	product price in the live-stream channel
λ	commission rate, $0 < \lambda < 1/2$
D_1	demand function of the direct sales channel
D_2	demand function of the live-streaming channel
Π_m	manufacturer's profit
Π_a	streamer's profit

3. MODELS UNDER DIFFERENT COMMISSION CONTRACTS

3.1. Pure Commission Contract (Model R)

In this structure, the manufacturer and the streamer adopt the pure commission mode. The market equilibrium demand function of the direct sales channel is given by

$$D_1=1-\alpha-p_1+\theta p_2$$

The market equilibrium demand function of the live-stream channel is given by

$$D_2 = \alpha - p_2 + \theta p_1 + e + \mu$$

The profit functions of the manufacturer and the streamer are as follows:

$$\Pi_m^R = p_1^R D_1^R + (1 - \lambda) p_2^R D_2^R \quad (1)$$

$$\Pi_a^R = \lambda p_2^R D_2^R - \frac{1}{2} (e^R)^2 \quad (2)$$

The two-stage structure adopts a Stackelberg game for decision-making, with the timing as follows: In the first stage, as the Stackelberg leader, the manufacturer determines the product price in its direct sales channel based on the profit maximization principle p_1^R ; In the second stage, the streamer formulates the selling price of the products p_2^R in the short video live streaming channel and the streamer's marketing efforts e^R based on the above decision-making. Using the backward induction method, the equilibrium solution can be obtained, as shown in Table 2.

Table 2. Model under the Pure Commission Contract (Model R)

$p_1^{R*} = \frac{(\lambda - 2)^2 (1 - \alpha) - \theta(3\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$p_2^{R*} = \frac{(\lambda - 2)(\alpha - 1)\theta - (\theta^2 + 2\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$e^{R*} = \frac{\lambda(\lambda - 2)(\alpha - 1)\theta - \lambda(\theta^2 + 2\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$\Pi_m^{R*} = \frac{(\lambda^2 + (6\theta - 8)\lambda + \theta^2 - 8\theta + 8)\alpha^2 + \left(\begin{array}{l} -2\lambda^2 + ((6\mu - 6)\theta - 8\mu + 8)\lambda \\ + 2\theta^2\mu + (-8\mu + 8)\theta + 8\mu - 8 \end{array} \right) \alpha + \lambda^2 + (-4\mu^2 - 6\mu\theta - 4)\lambda + \mu^2\theta^2 + 4\mu^2 + 8\mu\theta + 4}{8\lambda\theta^2 + 4\lambda^2 - 12\theta^2 - 16\lambda + 16}$
$\Pi_a^{R*} = \frac{(2 - \lambda) \left((\theta^2 + 2\lambda - 4)(\mu + \alpha) - (\lambda - 2)(\alpha - 1)\theta \right)^2 \lambda}{2 \left((4\lambda - 6)\theta^2 + 2(\lambda - 2)^2 \right)^2}$

Determine the sign of the denominator to identify whether it is positive or negative.

$(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2 = 2\lambda^2 + (4\theta^2 - 8)\lambda - 6\theta^2 + 8$, This expression can be treated as a quadratic function of λ , With a quadratic coefficient of 2 (positive), the function opens upward, and the linear coefficient is denoted as $4\theta^2 - 8$, the coefficients is denoted as $-6\theta^2 + 8$, and the discriminant is denoted as $\Delta = (4\theta^2 - 8)^2 - 8(-6\theta^2 + 8) = 16\theta^2(\theta^2 - 1) < 0$, The results indicate that the function opens upward and possesses no real roots. Consequently, we formally prove that the function is strictly positive, which verifies the inequality $(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2 > 0$.

Proposition 1. Under the pure commission contract regime, we have the following properties:

$$(i) \quad \frac{\partial p_1^{R*}}{\partial \mu} > 0, \quad \frac{\partial p_2^{R*}}{\partial \mu} > 0, \quad \frac{\partial e^{R*}}{\partial \mu} > 0;$$

$$(ii) \frac{\partial \Pi_m^{R^*}}{\partial \mu} > 0, \quad \frac{\partial \Pi_a^{R^*}}{\partial \mu} > 0.$$

$$\text{Proof: (i) } \frac{\partial p_1^{R^*}}{\partial \mu} = \frac{\theta(4-3\lambda)}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0, \quad \frac{\partial p_2^{R^*}}{\partial \mu} = \frac{4-\theta^2-2\lambda}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0,$$

$$\frac{\partial e^{R^*}}{\partial \mu} = \frac{\lambda(4-\theta^2-2\lambda)}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0;$$

$$(ii) \frac{\partial \Pi_m^{R^*}}{\partial \mu} = \frac{(4-3\lambda)(1-\alpha)\theta + (\mu+\alpha)(4+\theta^2-4\lambda)}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0,$$

$$\frac{\partial \Pi_a^{R^*}}{\partial \mu} = \frac{(2-\lambda)((\theta^2+2\lambda-4)(\mu+\alpha) - (\lambda-2)(\alpha-1)\theta)\lambda(\theta^2+2\lambda-4)}{4(2\lambda\theta^2 + \lambda^2 - 3\theta^2 - 4\lambda + 4)^2} > 0$$

Proposition 1 Under the pure commission contract regime, the following results hold:

(i) An increase in the streamer's influence leads to higher direct selling price, live-streaming price, and the streamer's marketing effort. Specifically, a stronger as a key exogenous driver of market demand and channel competitiveness—expands the demand scale and price space of the livestreaming channel, incentivizing the manufacturer to raise both direct and livestreaming prices while motivating the streamer to commit more marketing effort to enhance channel competitiveness and demand conversion rates.

(ii) Both the manufacturer's and the streamer's profits increase in the streamer's influence at the equilibrium. Intuitively, a higher the streamer's influence not only boosts the streamer's commission earnings directly but also expands the demand scale and profit margin of the direct-selling channel through the spillover effect, thereby achieving a simultaneous improvement in both parties' profits and contributing to a Pareto improvement in the overall supply chain system.

3.2. Commission-and-Service-Fee Contract (Model RI)

In this structure, the manufacturer and the streamer adopt the commission plus service fee mode. The market equilibrium demand function of the direct sales channel is given by

$$D_1 = 1 - \alpha - p_1 + \theta p_2 + \eta \mu$$

The market equilibrium demand function of the live-stream channel is given by

$$D_2 = \alpha - p_2 + \theta p_1 + e_1 + \mu$$

The profit functions of the manufacturer and the streamer are as follows:

$$\Pi_m^{RI} = p_1^{RI} D_1^{RI} + (1-\lambda) p_2^{RI} D_2^{RI} - \tau \mu \quad (3)$$

$$\Pi_a^{RI} = \lambda p_2^{RI} D_2^{RI} - \frac{1}{2} (e^{RI})^2 + \tau \mu \quad (4)$$

The two-stage structure adopts a Stackelberg game for decision-making, with the timing as follows: In the first stage, as the Stackelberg leader, the manufacturer determines the product price in its direct sales channel based on the profit maximization principle p_1^{RI} ; In the second stage, the streamer formulates the selling price of the products p_2^{RI} in the short video live streaming channel and the streamer's marketing efforts e^{RI} based on the above decision-making. Using the backward induction method, the equilibrium solution can be obtained, as shown in Table 3.

Table 3. Model under the Commission-and-Service-Fee Contract (Model RI)

$p_1^{RI*} = \frac{(\lambda - 2)^2 (\eta\mu - \alpha + 1) - \theta(3\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$p_2^{RI*} = \frac{(\lambda - 2)(\alpha - \eta\mu - 1)\theta - (\theta^2 + 2\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$e^{RI*} = \frac{\lambda(\lambda - 2)(\alpha - \eta\mu - 1)\theta - \lambda(\theta^2 + 2\lambda - 4)(\mu + \alpha)}{(4\lambda - 6)\theta^2 + 2(\lambda - 2)^2}$
$\Pi_m^{RI*} = \frac{(\eta^2\lambda^2 + (-4\eta^2 - 6\eta\theta - 4)\lambda + 4\eta^2 + 8\eta\theta + \theta^2 + 4)\mu^2 + \left[\begin{array}{l} (-2\alpha\eta + 2\eta - 4\tau)\lambda^2 + \left(\begin{array}{l} ((-6\eta + 6)\theta + 8\eta - 8)\alpha \\ -8\tau\theta^2 - 8\eta + 16\tau - 6\theta \end{array} \right) \lambda \\ + (2\theta^2 + (8\eta - 8)\theta - 8\eta + 8)\alpha + 12\tau\theta^2 + 8\eta - 16\tau + 8\theta \end{array} \right] \mu + (\alpha - 1)^2\lambda^2 + (-4 + (6\theta - 8)\alpha^2 + (-6\theta + 8)\alpha)\lambda + 4 + (\theta^2 - 8\theta + 8)\alpha^2 + (8\theta - 8)\alpha}{8\theta^2\lambda + 4\lambda^2 - 12\theta^2 - 16\lambda + 16}$
$\Pi_a^{RI*} = \frac{\lambda(2 - \lambda)\left((\theta^2 + 2\lambda - 4)(\mu + \alpha) - (\lambda - 2)(\alpha - \eta\mu - 1)\theta \right)^2}{2\left((4\lambda - 6)\theta^2 + 2(\lambda - 2)^2 \right)^2} + \tau\mu$

Proposition 1. Under the pure commission-and-service-fee contract regime, we have the following properties:

- (i) $\frac{\partial p_1^{RI*}}{\partial \mu} > 0$, $\frac{\partial p_2^{RI*}}{\partial \mu} > 0$, $\frac{\partial e^{RI*}}{\partial \mu} > 0$;
- (ii) $\frac{\partial \Pi_a^{RI*}}{\partial \mu} > 0$, when $0 < \tau < \tau_1$, then $\frac{\partial \Pi_m^{RI*}}{\partial \mu} > 0$; when $\tau_1 < \tau < 1$, then $\frac{\partial \Pi_m^{RI*}}{\partial \mu} < 0$.

Wherein,

$$\tau_1 = \frac{(\eta^2\mu + (-\alpha + 1)\eta)\lambda^2 + \left(\begin{array}{l} ((-6\mu - 3\alpha)\eta + 3\alpha - 3)\theta - 4\eta^2\mu \\ + (4\alpha - 4)\eta - 4\mu - 4\alpha \end{array} \right) \lambda + (\mu + \alpha)\theta^2 + ((8\mu + 4\alpha)\eta - 4\alpha + 4)\theta + 4\eta^2\mu + (-4\alpha + 4)\eta + 4\mu + 4\alpha}{2\lambda^2 + (4\theta^2 - 8)\lambda - 6\theta^2 + 8}$$

Proof: (i) $\frac{\partial p_1^{RI*}}{\partial \mu} = \frac{(\lambda-2)^2 \eta - \theta(3\lambda-4)}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0$, $\frac{\partial p_2^{RI*}}{\partial \mu} = \frac{(2-\lambda)\eta\theta - \theta^2 - 2\lambda + 4}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0$,

$$\frac{\partial e^{RI*}}{\partial \mu} = \frac{\lambda((2-\lambda)\eta\theta - \theta^2 - 2\lambda + 4)}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} > 0;$$

$$(ii) \quad \begin{aligned} & (-2\lambda^2 + (-4\theta^2 + 8)\lambda + 6\theta^2 - 8)\tau + (\eta^2\mu + (-\alpha + 1)\eta)\lambda^2 \\ & + \left(\begin{aligned} & ((-6\mu - 3\alpha)\eta + 3\alpha - 3)\theta - 4\eta^2\mu \\ & + (4\alpha - 4)\eta - 4\mu - 4\alpha \end{aligned} \right) \lambda \\ & + (\mu + \alpha)\theta^2 + ((8\mu + 4\alpha)\eta - 4\alpha + 4)\theta + 4\eta^2\mu \\ & \frac{\partial \Pi_m^{RI*}}{\partial \mu} = \frac{+(-4\alpha + 4)\eta + 4\mu + 4\alpha}{(4\lambda-6)\theta^2 + 2(\lambda-2)^2} \end{aligned} \quad (5)$$

Treat the numerator of Equation (5) as a linear function of τ , and it is easy to prove that the coefficient of the linear term is less than 0, so this function is a decreasing function of τ .

When $\tau = 0$, the numerator expression of Equation (5) is:

$$(ii) \quad \begin{aligned} & (\eta^2\mu + (-\alpha + 1)\eta)\lambda^2 + \left(\begin{aligned} & ((-6\mu - 3\alpha)\eta + 3\alpha - 3)\theta \\ & - 4\eta^2\mu + (4\alpha - 4)\eta - 4\mu - 4\alpha \end{aligned} \right) \lambda \\ & + (\mu + \alpha)\theta^2 + ((8\mu + 4\alpha)\eta - 4\alpha + 4)\theta + 4\eta^2\mu + (-4\alpha + 4)\eta + 4\mu + 4\alpha \end{aligned} \quad (6)$$

Treat equation (5) as a quadratic function of τ ; the quadratic coefficient is positive, and its axis of symmetry is:

$$\frac{4\eta^2\mu + ((3\theta - 4)\alpha + 6\theta\mu + 4)\eta + (-3\theta + 4)\alpha + 4\mu + 3\theta}{2\eta(\eta\mu - \alpha + 1)}$$

Since, $\frac{4\eta^2\mu + ((3\theta - 4)\alpha + 6\theta\mu + 4)\eta + (-3\theta + 4)\alpha + 4\mu + 3\theta}{2\eta(\eta\mu - \alpha + 1)} - \frac{1}{2}$
 $= \frac{4\eta^2\mu + \alpha(3\theta - 4)(\eta - 1) + (6\theta\mu + 4)\eta + 4\mu + 3\theta}{2\eta(\eta\mu - \alpha + 1)} > 0$

The axis of symmetry of equation (5) exceeds $\frac{1}{2}$. Thus, equation (6) is a quadratic function in λ opening upward, maximized at $\lambda = 0$ and minimized at $\lambda = \frac{1}{2}$.

Let $G_1(\lambda) = (\eta^2\mu + (-\alpha + 1)\eta)\lambda^2 + \left(\begin{aligned} & ((-6\mu - 3\alpha)\eta + 3\alpha - 3)\theta \\ & - 4\eta^2\mu + (4\alpha - 4)\eta - 4\mu - 4\alpha \end{aligned} \right) \lambda$
 $+ (\mu + \alpha)\theta^2 + ((8\mu + 4\alpha)\eta - 4\alpha + 4)\theta + 4\eta^2\mu + (-4\alpha + 4)\eta + 4\mu + 4\alpha$

$$G_1\left(\frac{1}{2}\right) = (\mu + \alpha)\theta^2 + \frac{5((\eta - 1)\alpha + 2\eta\mu + 1)\theta}{2} + \frac{(-9\eta + 8)\alpha}{4} + \frac{9\eta^2\mu}{4} + 2\mu + \frac{9\eta}{4} > 0$$

$$G_1(\lambda) > G_1\left(\frac{1}{2}\right) > 0$$

When $\tau = 0$, then $\frac{\partial \Pi_m^{RI*}}{\partial \mu} > 0$; when $\tau = 1$, the sign of the function is indefinite so it depends on τ .

Set Equation (5) equal to 0 to obtain the threshold τ_1 . When $0 < \tau < \tau_1$, then $\frac{\partial \Pi_m^{RI*}}{\partial \mu} > 0$; when

$$\tau_1 < \tau < 1, \quad \frac{\partial \Pi_m^{RI*}}{\partial \mu} < 0.$$

$$\tau_1 = \frac{(\eta^2\mu + (-\alpha + 1)\eta)\lambda^2 + \left(\begin{array}{l} ((-6\mu - 3\alpha)\eta + 3\alpha - 3)\theta - 4\eta^2\mu \\ + (4\alpha - 4)\eta - 4\mu - 4\alpha \end{array} \right)\lambda}{2\lambda^2 + (4\theta^2 - 8)\lambda - 6\theta^2 + 8} + \frac{(\mu + \alpha)\theta^2 + ((8\mu + 4\alpha)\eta - 4\alpha + 4)\theta + 4\eta^2\mu + (-4\alpha + 4)\eta + 4\mu + 4\alpha}{2\lambda^2 + (4\theta^2 - 8)\lambda - 6\theta^2 + 8}$$

$$\frac{\partial \Pi_a^{RI*}}{\partial \mu} = \frac{\lambda(2 - \lambda)\left((\theta^2 + 2\lambda - 4)(\mu + \alpha) - (\lambda - 2)(-\mu\eta + \alpha - 1)\theta\right) + (\theta^2 + 2\lambda - 4 + (\lambda - 2)\eta\theta)}{\left((4\lambda - 6)\theta^2 + 2(\lambda - 2)^2\right)^2} + \tau > 0$$

Proposition 2 shows that:

(i) Under the commission-plus-service-fee contract, higher streamer influence significantly raises the direct selling price, live-stream price, and streamer marketing effort. The service fee strengthens the streamer's income security and cooperation incentives. As streamer influence grows, the live-stream channel becomes more attractive, prompting higher marketing effort to boost sales and driving up both channel prices, consistent with the pure commission contract.

(ii) Under the commission-plus-service-fee contract, streamer influence always increases the streamer's profit, but its effect on the manufacturer's profit depends on the service fee coefficient: the manufacturer's profit rises with streamer influence when the service fee coefficient is low, and falls when the coefficient exceeds a threshold. The service fee coefficient is the key threshold: low fees let demand growth and spillover gains cover costs, while excessive fees erode earnings, turning the positive effect of streamer influence negative. Thus, the service fee should be set within a reasonable range to maximize the manufacturer's profit while expanding the market via streamer influence.

Proposition 3 Under the pure commission-and-service-fee contract regime, we have the following properties:

$$(i) \quad \frac{\partial p_1^{RI*}}{\partial \eta} > 0, \quad \frac{\partial p_2^{RI*}}{\partial \eta} > 0, \quad \frac{\partial e^{RI*}}{\partial \eta} > 0;$$

$$(ii) \frac{\partial \Pi_m^{RI*}}{\partial \eta} > 0, \frac{\partial \Pi_a^{RI*}}{\partial \eta} > 0.$$

$$\text{Proof: (i) } \frac{\partial p_1^{RI*}}{\partial \eta} = \frac{(\lambda-2)^2 \mu}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0, \quad \frac{\partial p_2^{RI*}}{\partial \eta} = \frac{(2-\lambda)\mu\theta}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0,$$

$$\frac{\partial e^{RI*}}{\partial \eta} = \frac{\lambda(2-\lambda)\mu\theta}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0;$$

$$(ii) \frac{\partial \Pi_m^{RI*}}{\partial \eta} = \frac{\mu((\mu\eta - \alpha + 1)\lambda^2 + 4(1-\lambda)(\mu\eta - \alpha + 1) + \theta(4-3\lambda)(\mu + \alpha))}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0,$$

$$\frac{\partial \Pi_a^{RI*}}{\partial \eta} = \frac{\lambda(2-\lambda)((\theta^2 + 2\lambda - 4)(\mu + \alpha) - (\lambda-2)(-\mu\eta + \alpha - 1)\theta)(\lambda-2)\mu\theta}{((4\lambda-6)\theta^2 + 2(\lambda-2)^2)^2} > 0.$$

Proposition 3 shows that:

(i) Under the commission-plus-service-fee contract, a higher spillover coefficient raises the direct price, live-stream price and streamer marketing effort. A stronger spillover boosts brand exposure and traffic diversion to the direct channel, expanding potential demand. The manufacturer then sets a higher direct price for higher marginal profit, while the streamer raises the live-stream price and exerts more effort to improve conversion.

(ii) A higher spillover coefficient increases both the manufacturer's and streamer's profits, achieving win-win performance. The spillover expands sales and price margins for both channels. Therefore, unlocking positive channel spillovers is key to synergy and mutual profit in dual-channel supply chains.

3.3. Comparative Analysis of the Two Models

3.3.1. Comparison of Product Prices

Proposition 4. Under different contracts, the direct selling prices satisfy: $p_1^{RI*} > p_1^{R*}$.

Proof:

$$p_1^{RI*} - p_1^{R*} = \frac{\eta\mu(\lambda-2)^2}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0$$

Proposition 4 shows that the direct selling price is higher under the commission-plus-service-fee contract than under the pure commission contract. The service fee gives the manufacturer rights to reuse live content and spillover gains, supporting a higher direct price. Meanwhile, brand and demand effects from streamer influence further lift the direct price. Thus, the service-fee mechanism drives up the direct price through both revenue structure and market demand.

Proposition 5 Under different contracts, the live-stream prices satisfy: $p_2^{RI*} > p_2^{R*}$.

Proof:

$$p_2^{RI*} - p_2^{R*} = \frac{\eta\mu\theta(2-\lambda)}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0$$

Proposition 5 shows the live-stream price is higher under the commission-plus-service-fee contract. The service fee provides stable income for the streamer and enhances marketing incentives, leading

to a higher live-stream price. Dual-channel price linkage also creates room for price increase. The hybrid contract raises the equilibrium live-stream price through incentives and price transmission.

3.3.2. Comparison of Streamer Marketing Effort

Proposition 6 Under different contracts, the streamer marketing effort satisfy: $e^{RI^*} > e^{R^*}$.

Proof:

$$e^{RI^*} - e^{R^*} = \frac{\lambda\eta\mu\theta(2-\lambda)}{4\lambda\theta^2 + 2\lambda^2 - 6\theta^2 - 8\lambda + 8} > 0$$

Proposition 6 shows streamer effort is higher under the commission-plus-service-fee contract. This contract provides stronger incentives for the streamer. The service fee reduces income risk and boosts enthusiasm for live marketing, content promotion and fan operation. Channel spillover effects further motivate greater effort. Thus, the hybrid contract more effectively stimulates marketing input and improves live-stream channel efficiency.

3.3.3. Comparison of Profits

Proposition 4. Under different contracts, the profit of the manufacturer satisfy: when $0 < \tau < \tau_2$, then $\Pi_m^{RI^*} > \Pi_m^{R^*}$; when $\tau_2 < \tau < 1$, then $\Pi_m^{RI^*} < \Pi_m^{R^*}$.

Wherein, $\tau_2 = \frac{\eta((\eta\mu - 2\alpha + 2)\lambda^2 + ((-6\mu - 6\alpha)\theta - 4\eta\mu + 8\alpha - 8)\lambda + (8\mu + 8\alpha)\theta + 4\eta\mu - 8\alpha + 8)}{8\lambda\theta^2 + 4\lambda^2 - 12\theta^2 - 16\lambda + 16}$

Proof:

$$\Pi_m^{RI^*} - \Pi_m^{R^*} = \frac{\mu \left[\begin{aligned} &((-4\lambda^2 + (-8\theta^2 + 16)\lambda + 12\theta^2 - 16)\tau \\ &+ (\eta^2\mu - 2\alpha\eta + 2\eta)\lambda^2 + ((-6\theta + 8)\eta\alpha - 4\eta^2\mu + (-6\mu\theta - 8)\eta)\lambda \\ &+ (8\theta - 8)\eta\alpha + 4\eta^2\mu + (8\mu\theta + 8)\eta \end{aligned} \right]}{8\lambda\theta^2 + 4\lambda^2 - 12\theta^2 - 16\lambda + 16}$$

Given that the denominator is positive, the numerator may be expressed as $\mu G(\tau)$,

$G(\tau)$ is a linear function of τ with a negative coefficient for the linear term of τ , thus, $G(\tau)$ is a strictly decreasing linear function of τ . When $\tau = 0$, part of the numerator can be simplified as:

$$(\eta^2\mu - 2\alpha\eta + 2\eta)\lambda^2 + ((-6\theta + 8)\eta\alpha - 4\eta^2\mu + (-6\mu\theta - 8)\eta)\lambda + (8\theta - 8)\eta\alpha + 4\eta^2\mu + (8\mu\theta + 8)\eta$$

Let $G_2(\lambda) = (\eta^2\mu - 2\alpha\eta + 2\eta)\lambda^2 + ((-6\theta + 8)\eta\alpha - 4\eta^2\mu + (-6\mu\theta - 8)\eta)\lambda + (8\theta - 8)\eta\alpha + 4\eta^2\mu + (8\mu\theta + 8)\eta$

$G_2(\lambda)$ is a quadratic function of λ , the coefficient of the quadratic term is greater than 0, and the axis of symmetry is:

$$\frac{(3\theta - 4)\alpha + 4 + (2\eta + 3\theta)\mu}{\eta\mu - 2\alpha + 2} > 0$$

Within the feasible domain, the upper bound of λ is $\frac{1}{2}$,

$$\frac{(3\theta-4)\alpha+4+(2\eta+3\theta)\mu}{\eta\mu-2\alpha+2} - \frac{1}{2} = \frac{(6\theta-6)\alpha+6+(3\eta+6\theta)\mu}{2\eta\mu-4\alpha+4} > 0$$

Thus, $G_2(\lambda)$ is a convex quadratic function whose axis of symmetry is greater than $\frac{1}{2}$, maximized at $\lambda=0$ and minimized at $\lambda=\frac{1}{2}$.

$$G_2(\lambda) > G_2\left(\frac{1}{2}\right) = \frac{\eta((9\eta+20\theta)\mu+20\alpha\theta-18\alpha+18)}{4} > 0$$

When $\tau=0$, then $\Pi_m^{RI*} - \Pi_m^{R*} > 0$; when $\tau=1$, the sign of the function is indefinite so it depends on τ .

Letting $\Pi_m^{RI*} - \Pi_m^{R*} = 0$, we obtain the threshold τ_2 . When $0 < \tau < \tau_2$, $\Pi_m^{RI*} - \Pi_m^{R*} > 0$; when $\tau_2 < \tau < 1$, $\Pi_m^{RI*} - \Pi_m^{R*} < 0$.

$$\text{Wherein, } \tau_2 = \frac{\eta((\eta\mu-2\alpha+2)\lambda^2 + ((-6\mu-6\alpha)\theta - 4\eta\mu + 8\alpha - 8)\lambda + (8\mu+8\alpha)\theta + 4\eta\mu - 8\alpha + 8)}{8\lambda\theta^2 + 4\lambda^2 - 12\theta^2 - 16\lambda + 16}.$$

Proposition 7 shows that under the commission-plus-service-fee contract, the manufacturer achieves a higher profit when the service fee coefficient is low, whereas the pure commission contract yields a higher manufacturer profit once the service fee coefficient exceeds the critical threshold. The service fee coefficient serves as the pivotal threshold variable governing the manufacturer's contract choice. When the service fee is relatively low, the benefits from spillover gains, improved marketing effort, and demand expansion outweigh the associated cost, thereby boosting the manufacturer's profit. By contrast, an excessively high service fee leads to mounting costs that offset the incremental revenue from increased demand and prices, reducing the manufacturer's profit.

Proposition 8. Under different contracts, the streamer's profit satisfy: $\Pi_a^{RI*} > \Pi_a^{R*}$.

Proof:

$$\Pi_a^{RI*} - \Pi_a^{R*} = \frac{\lambda(2-\lambda)^2(2(4-\theta^2-2\lambda)(\mu+\alpha) + (2-\lambda)(2-2\alpha+\eta\mu)\theta)\mu\theta\eta}{2((4\lambda-6)\theta^2 + 2(\lambda-2)^2)^2} + \tau\mu > 0$$

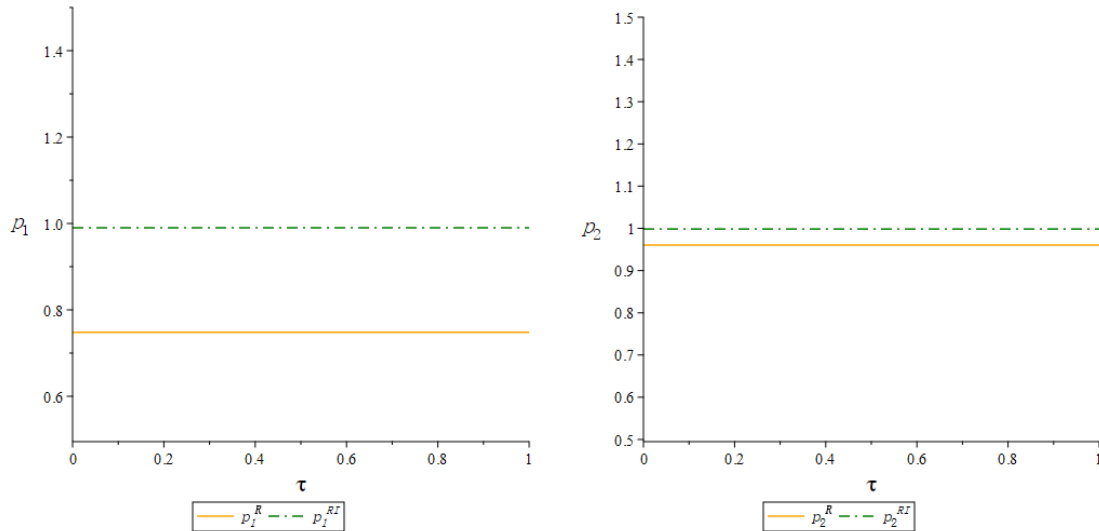
Proposition 8 shows that the streamer's profit under the commission-plus-service-fee contract is strictly higher than that under the pure commission contract. The streamer obtains higher and more stable income under the commission-plus-service-fee contract. The service fee provides an additional revenue stream for the streamer, while higher marketing effort and live-stream selling prices further expand commission income. Driven by these two factors, the streamer's profit improves significantly. This implies that the streamer naturally prefers the commission-plus-service-fee cooperation model, and this conclusion can provide an important basis for supply chain negotiation, profit allocation, and contract coordination.

3.4. Numerical Example and Sensitivity Analysis

We conduct numerical simulations to examine equilibrium decisions under two commission contracts: pure commission and commission-plus-service-fee. We focus on how the service fee coefficient τ affects the overall supply chain system.

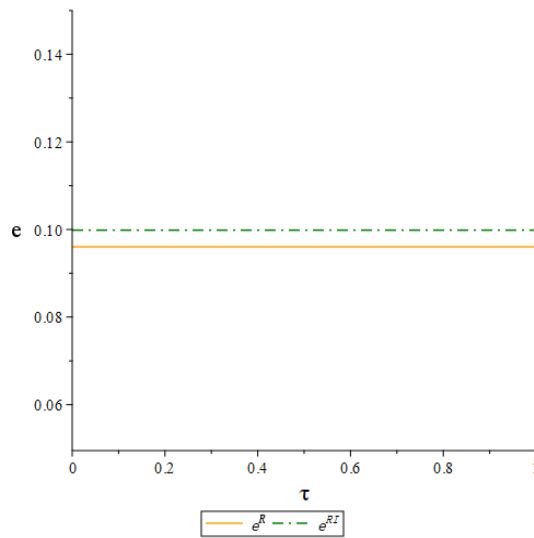
Following existing literature [7, 10-12], we set the relevant parameters as follows: $\alpha = 0.1$, $\theta = 0.3$, $\eta = 0.3$, $\mu = 0.15$, $\tau = 0.3$, $f = 0.5$, $k = 0.5$, $\lambda = 0.1$.

The mechanism by which the service fee coefficient τ affects the decision variables and profits of supply chain members is illustrated by the empirical results in figure 2:



(a) Impact of τ on the direct-selling price

(b) Impact of τ on the live-stream price



(c) Impact of τ on the streamer's marketing effort

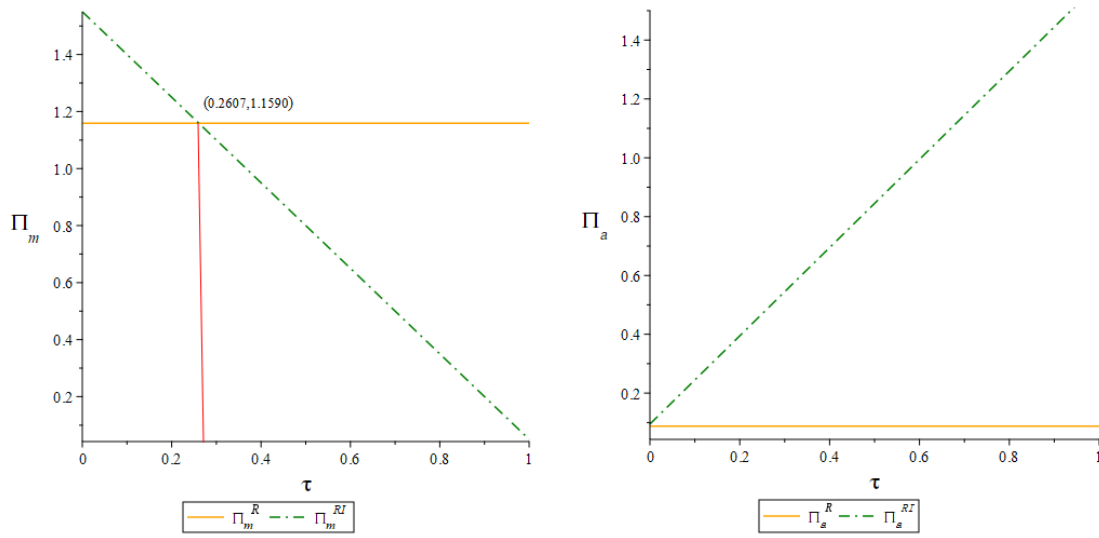
Figure 2. Impact of τ on the optimal decisions of supply chain members

Based on the empirical results shown in Figures 2(a)-2(c), the two commission modes in the live-stream e-commerce supply chain affect supply chain decisions as follows:

- (i) The direct-selling price: $p_1^{RI*} > p_1^{R*}$;
- (ii) The live-stream price: $p_2^{RI*} > p_2^{R*}$;

(iii) The streamer marketing effort: $e^{RI^*} > e^{R^*}$.

It is evident that the service fee coefficient exerts a significant influence on the optimal supply chain decisions under both contract schemes. Specifically, compared with the pure commission contract (Model R), the direct selling price, livestream price, and the streamer's marketing effort level are consistently higher in the commission-plus-service-fee contract (Model RI). This finding is consistent with Propositions 4, 5, and 6. As the service fee coefficient increases, the decision variables under both contract schemes exhibit an upward trend, and the commission-plus-service-fee contract exerts a more pronounced stimulating effect on prices and effort. This result validates the theoretical analysis: the service fee effectively strengthens the streamer's marketing incentives, expands the space for channel demand, and drives simultaneous increases in both direct and livestream selling prices as well as marketing effort levels. Overall, it indicates that the commission-plus-service-fee contract outperforms the pure commission contract in incentivizing the streamer's investment and enhancing the channel's pricing power.



(a) Impact of τ on the manufacturer's profit (b) Impact of τ on the streamer's profit

Figure 3. Impact of τ on the Profits of Supply Chain Member

Based on the empirical results shown in Figures 3(a)-3(b), the two commission modes in the live-stream e-commerce supply chain affect supply chain profits as follows:

(i) Manufacturer's profit: when $0 < \tau < \tau_2$, $\Pi_m^{RI^*} > \Pi_m^{R^*}$; when $\tau_2 < \tau < 1$, $\Pi_m^{RI^*} < \Pi_m^{R^*}$;

(ii) Streamer's profit: $\Pi_a^{RI^*} > \Pi_a^{R^*}$.

From the above analysis, it can be concluded that under the commission-plus-service-fee contract, the streamer's profits consistently dominate and continue to grow as the service fee coefficient (τ) increases. In contrast, the manufacturer's profits exhibit a clear threshold effect and an intersection point characteristic. Specifically, when the service fee coefficient is low, the commission-plus-service-fee contract is more beneficial for the manufacturer's profitability. When the service fee coefficient reaches a specific threshold ($\tau = 0.2607$), the manufacturer's profits under both contracts are equal, with the intersection profit being 1.1590. Once the service fee coefficient exceeds this critical value, the manufacturer's profits under the pure commission contract become higher. This finding is consistent with those presented in Propositions 7 and 8.

4. SUMMARY

This paper This chapter focuses on a dual-channel supply chain in live-stream e-commerce, featuring a manufacturer with direct-to-consumer (D2C) operations integrated with live-streaming sales. We construct two Stackelberg game models under distinct contract structures: a pure commission model (Model R) and a commission-plus-service-fee model (Model RI). Using backward induction to derive equilibrium solutions, we investigate the impacts of streamer influence, channel spillover effects, and the service fee coefficient on dual-channel pricing, the streamer's marketing effort, and the profits of supply chain members. Theoretical conclusions are further validated through numerical examples. The key findings are as follows:

- (i) **Streamer Influence:** Regardless of the contract scheme, an increase in streamer influence significantly boosts dual-channel pricing and the streamer's marketing effort. This leads to a simultaneous growth in profits for both the manufacturer and the streamer, exerting a strong positive driving effect on the overall supply chain system.
- (ii) **Threshold Effect of Service Fee Coefficient:** Under the commission-plus-service-fee contract, the service fee coefficient exhibits a threshold effect on manufacturer profits. When the coefficient is low, streamer influence enhances manufacturer profitability. However, once the coefficient exceeds a critical threshold, the cost erosion effect dominates, causing manufacturer profits to decline as streamer influence increases.
- (iii) **Channel Spillover Effects:** Positive channel spillover effects effectively expand demand across both channels, simultaneously increasing channel pricing, streamer effort, and member profits. This constitutes a critical factor in achieving collaborative value enhancement across the supply chain.
- (iv) **Cross-Model Comparison:** A horizontal comparison of the two contract models reveals that the commission-plus-service-fee model yields higher dual-channel prices, greater streamer effort, and elevated streamer profits compared to the pure commission model. However, manufacturer profits are constrained by the service fee coefficient. The commission-plus-service-fee model only demonstrates superior advantages when the service fee falls within a reasonable range.

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