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Joint Replenishment Problems in Inventory and Distribution Systems: A Review of Models, Constraints, and Algorithms

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ABSTRACT

The joint replenishment problem (JRP) examines replenishment cycles, order quantities, and coordinated execution decisions for multiple items that share ordering costs or operational resources. As inventory management becomes increasingly connected with distribution networks, JRP has moved beyond static cost minimization and has been extended to stochastic demand, dynamic demand, resource constraints, joint replenishment and delivery, inventory routing, vendor-managed inventory, and learning-based optimization. This paper provides a narrative review of representative studies on JRP and related integrated inventory-distribution problems. The review is organized around four themes: relaxation of modeling assumptions, incorporation of operational constraints, expansion of decision boundaries, and evolution of algorithmic paradigms. It compares the decision scope of JRP, JRD, IRP, and S&OP-related models, and discusses the applicability of heuristics, metaheuristics, mathematical programming, approximation algorithms, and reinforcement learning. The review suggests that JRP research is shifting from replenishment-cycle selection under a single cost objective toward integrated supply chain decision-making under uncertain demand, limited resources, distribution coordination, and multi-objective performance requirements.

KEYWORDS

Joint replenishment problem; Joint replenishment and delivery; Heuristic algorithms; Reinforcement learning

1. INTRODUCTION

In multi-item inventory systems, different items often share fixed costs associated with ordering, shipment preparation, information processing, warehousing, or transportation. If each item is replenished independently, fixed costs may be paid repeatedly. If all items are replenished at the same time without discrimination, the system may save ordering cost but accumulate unnecessary inventory. The joint replenishment problem (JRP) addresses this trade-off by coordinating replenishment cycles and order quantities across multiple items.

The classical JRP provides a compact way to explain the economic value of coordination. It extends the economic order quantity logic from a single item to multiple items that share a major ordering cost. The key managerial question is not whether all items should always be synchronized, but how much synchronization is economically justified when demand rates, minor ordering costs, and holding costs differ across items.

Early reviews have established the main theoretical foundation of this field. Khouja and Goyal reviewed the JRP literature from 1989 to 2005 and summarized the development of classical models, solution approaches, and extensions [1]. Peng et al. further reviewed studies from 2006 to 2022 and organized the literature into stochastic demand, dynamic demand, resource constraints, special extensions, and joint replenishment and delivery (JRD) [2]. These review frameworks remain useful, but recent research has increasingly connected JRP with sales and operations planning (S&OP), inventory routing, vendor-managed inventory (VMI), robust optimization, and learning-based decision-making.

This paper is written as a narrative review for inventory and distribution management. Instead of arranging the literature only by publication year, it synthesizes representative studies around four questions: which assumptions of the classical model have been relaxed; which operational constraints have been introduced; how the decision boundary has expanded from replenishment to distribution and routing; and why different algorithmic paradigms are used under different problem structures. The purpose is to provide a clear and practically oriented map of JRP research for readers interested in supply chain coordination and optimization.

2. LITERATURE SOURCES AND CLASSIFICATION FRAMEWORK

This review is based on representative English-language studies collected from major academic databases and publisher platforms, including Web of Science, ScienceDirect, SpringerLink, Taylor & Francis, Wiley, Google Scholar, and arXiv. The literature covers the period from the late 1980s to recent studies. Search terms included joint replenishment problem, joint replenishment and delivery, stochastic joint replenishment, dynamic-demand joint replenishment, resource-constrained joint replenishment, inventory routing problem, vendor-managed inventory, and reinforcement learning inventory replenishment.

The selected studies follow three principles. First, the study should explicitly address JRP, JRD, or closely related integrated inventory-distribution problems. Second, it should contain substantive content on modeling assumptions, optimization methods, performance evaluation, or application settings. Third, studies that help explain the expansion of the research boundary, such as S&OP integration, VMI, IRP, and learning-based optimization, are retained even when they are not traditional JRP papers.

To support comparison, the literature is organized along four dimensions: demand characteristics, constraint types, decision boundaries, and algorithmic paradigms. This classification is suitable for a narrative review because it highlights the evolution of problem structure rather than merely listing papers by time.

Table 1. Literature classification framework

Dimension	Classification criterion	Related research content
Demand characteristics	Whether demand is deterministic, stochastic, time-varying, or distribution-dependent.	Classical JRP, stochastic JRP, dynamic-demand JRP, stochastic and dynamic IRP.
Constraint types	Whether the model includes capacity, budget, shipment, minimum order quantity, service level, or time-varying cost constraints.	Resource-constrained JRP, CDJRP-TC, robust and multi-objective models.
Decision boundaries	Whether the model determines only replenishment decisions or also delivery, routing, warehousing, and planning decisions.	JRP, JRD, IRP, S&OP, VMI.
Algorithmic paradigms	Whether the solution approach relies on exact methods, heuristics, metaheuristics, approximation algorithms, or learning-based optimization.	Dynamic programming, MILP, genetic algorithms, differential evolution, PPO, GNN, approximation policies.

3. BASIC MODEL AND CLASSICAL STRATEGIES

The classical JRP can be viewed as an extension of the economic order quantity model to a multi-item system. It usually assumes that demand rates are known and stable, shortages and quantity discounts are not considered, holding costs are linear in inventory levels, and all items share a major ordering cost. Let S denote the major ordering cost, s_i the minor ordering cost of item i, d_i the demand rate, h_i the holding cost per unit, T the basic replenishment cycle, and k_i an integer multiplier for item i.

Under the indirect grouping strategy (IGS), the replenishment cycle and order quantity of item i can be expressed as:

$$T_i = k_i, \; Q_i = d_i k_i T \tag{1}$$

The corresponding average total cost is commonly written as:

$$TC(T,k) = \frac{S}{T} + \sum_i s_i / k_i T + \frac{T}{2} \sum_i d_i k_i h_i \tag{2}$$

Equation (2) shows the basic trade-off in JRP. A smaller basic cycle reduces average inventory but increases the frequency of major ordering. A larger basic cycle reduces shared ordering costs but increases holding costs, especially for items with high demand rates or high holding costs. Therefore, the optimization problem combines continuous cycle selection with integer multiplier selection.

Two classical coordination policies are indirect grouping strategy and direct grouping strategy (DGS). IGS derives item cycles from a common basic cycle and integer multipliers, while DGS first partitions items into groups and then assigns replenishment cycles to each group. Van Eijs et al. compared the two strategies and showed that the relative performance depends on the cost structure, particularly the magnitude of the major ordering cost [3]. When the major ordering cost is large, policies that encourage more joint orders may be more attractive. When holding cost differences dominate, excessive synchronization may be inefficient.

The classical model remains important because many later extensions can be interpreted as relaxations of its assumptions. Stochastic demand relaxes demand certainty, dynamic demand relaxes time invariance, resource-constrained models relax unlimited capacity, and JRD or IRP models expand the decision boundary from replenishment to distribution execution.

Figure 1. Evolutionary logic of joint replenishment research. Source: author's compilation

4. MAIN RESEARCH STREAMS

4.1. Stochastic Demand JRP

The stochastic joint replenishment problem (SJRP) treats demand as an uncertain stochastic process. Compared with the deterministic model, the focus moves from simply selecting replenishment cycles to controlling cost and service risk. Ozkaya et al. proposed a stochastic replenishment policy and analyzed its behavior under Poisson demand [4]. Viswanathan studied lower-bound selection for the stochastic JRP [5]. Kayis et al. modeled a two-item can-order policy as a semi-Markov decision process [6]. These studies introduced demand uncertainty into the replenishment trigger mechanism, although they often require explicit assumptions about demand distribution or state transition behavior.

Subsequent studies incorporated more realistic features, including cyclic policies, controllable lead times, mixtures of backorders and lost sales, and random numbers of imperfect items [7-9]. These extensions make SJRP closer to operational inventory risk, but they also increase parameter requirements and computational difficulty. In practice, the value of an SJRP model depends not only on mathematical elegance but also on whether demand distributions, service targets, and lead-time parameters can be estimated with acceptable reliability.

A useful distinction can be made between policy design and policy evaluation. Policy design focuses on how to define ordering triggers, review intervals, and joint replenishment conditions. Policy evaluation focuses on whether the resulting policy remains stable under misspecified demand distributions or changing service requirements. Recent inventory systems often face intermittent demand, promotion effects, and demand shocks, so the robustness of stochastic replenishment policies remains a relevant issue.

4.2. Dynamic Demand JRP and Integrated Planning

The dynamic-demand joint replenishment problem (DJRP) assumes that demand varies over a planning horizon but can be forecast or represented by period-specific values. Unlike the deterministic static JRP, DJRP must coordinate replenishment decisions across both item and time dimensions. Federgruen et al. proposed progressive interval heuristics for multi-item capacitated lot-sizing problems [10]. Robinson et al. developed effective heuristics for the dynamic-demand JRP [11], and later reviewed models and algorithms for coordinated deterministic dynamic lot-sizing [12]. Narayanan and Robinson evaluated joint replenishment procedures in rolling horizon planning systems [13].

In more complex dynamic-demand environments, storage capacity, discrete lot sizes, and transportation costs strongly affect replenishment timing. Gutierrez et al. studied replenishment policies for the multi-item dynamic lot-sizing problem with storage capacities [14]. Gicquel and Minoux developed valid inequalities for discrete lot-sizing and scheduling [15]. Baller et al. further considered dynamic-demand joint replenishment with approximated transportation costs [16]. These studies show that the difficulty of DJRP does not come only from changing demand, but from the interaction between time-varying requirements, inventory capacity, and transport-related costs.

A recent development is the connection between DJRP and S&OP. Suemitsu et al. introduced a dynamic-demand JRP with time-varying costs and transportation resource constraints, linking sales planning, inventory targets, and transportation capacity in a single planning context [17]. This direction is important because replenishment decisions in actual firms are rarely made as isolated inventory calculations. They are usually embedded in rolling plans, budget constraints, transport procurement, and service-level commitments.

4.3. Resource-Constrained JRP

Resource-constrained JRP introduces storage capacity, transportation capacity, budget, minimum order quantities, shipment constraints, or vehicle availability into the joint replenishment decision. Porras and Dekker proposed an efficient solution method for the JRP with minimum order quantities [18]. Hoque developed a model considering storage and transport capacities together with budget constraints [19]. Moon and Cha analyzed JRP under resource restrictions and compared RAND and genetic algorithm approaches [20]. These studies demonstrate that a cost-optimal replenishment schedule may not be implementable when operational resources are limited.

As the number of constraints increases, the problem often shifts from cycle optimization to large-scale combinatorial optimization. Chen et al. considered multiple trucks with shipment and resource constraints [21]. Ongkunaruk et al. incorporated defective items and shipment restrictions into a genetic algorithm framework [22]. Muriel et al. further improved algorithms for the JRP with minimum order quantities [23]. These studies suggest that resource-constrained JRP is not merely a minor modification of the classical model. Resource limits change the feasible region and may alter the economic value of synchronization itself.

For managers, resource constraints are often more visible than cost functions. Warehouses have finite space, transportation fleets have finite capacity, and suppliers may impose minimum order quantities. Therefore, resource-constrained JRP is particularly relevant when replenishment decisions must be converted into executable operational plans.

4.4. Special Extensions and Algorithmic Development

Special extensions of JRP mainly concern price mechanisms, product attributes, information errors, and inventory control policies. Moon et al. studied a JRP involving multiple suppliers offering quantity discounts and used a hybrid genetic algorithm [24]. Wang and Cheng analyzed the sensitivity of RAND-based solutions under inaccurate holding cost estimates and demand forecasts [25]. Tsao and Teng studied the joint multi-item replenishment problem under trade credits [26]. These studies are valuable because they convert realistic commercial conditions into model elements that influence replenishment coordination.

From an algorithmic perspective, JRP has long relied on heuristics and metaheuristics. Genetic algorithms, differential evolution, simulated annealing, variable neighborhood search, and hybrid methods are attractive because they can handle nonlinear cost structures, integer variables, and multiple constraints. However, their performance may depend on parameter settings, instance structure, and termination rules. For this reason, algorithmic research increasingly needs to report not only objective values but also stability, computational effort, and managerial interpretability.

Two newer routes have become visible. The first is learning-based optimization. Vanvuchelen et al. applied proximal policy optimization to the JRP and showed how reinforcement learning can generate replenishment policies under repeated decision settings [27]. The second is approximation-oriented analysis. Segev studied the continuous-time JRP and developed epsilon-optimal policies via pairwise alignment [28]. These routes address different questions: learning-based methods ask whether policies can adapt from state information, while approximation methods ask whether solution quality can be theoretically bounded.

4.5. JRD, IRP, and Integrated Replenishment-Distribution Decisions

JRD places joint replenishment and outbound delivery in a single model. A typical structure includes a supplier, a central warehouse, and multiple retailers or terminal nodes. Cha et al. studied joint replenishment and delivery scheduling in a one-warehouse, multi-retailer system [29]. Wang et al. proposed a differential evolution algorithm for an integrated stochastic replenishment and delivery model [30]. Qu et al. modeled and optimized JRD with heterogeneous items [31]. Cui et al. further studied a joint replenishment and synthetic delivery problem in a warehouse-centralized supply chain [32]. Zeng et al. developed a hybrid differential evolution and simulated annealing algorithm for JRD with trade credit [33].

The related inventory routing problem (IRP) further integrates replenishment quantities, vehicle routes, and delivery schedules. Coelho et al. reviewed thirty years of IRP research and showed that the integration of inventory and routing decisions can reduce system-level costs but increases computational complexity [34]. In a more recent VMI setting, Lu et al. proposed enhanced multi-task deep reinforcement learning for the integrated inventory-routing problem [35]. Although IRP is not identical to traditional JRP, it reflects the same coordination logic: replenishment decisions should be evaluated together with distribution execution when transportation and service requirements are significant.

The practical value of JRD and IRP lies in their ability to link inventory decisions with logistics execution. Traditional JRP may recommend economical replenishment cycles, but these cycles can be difficult to implement if vehicle routes, delivery time windows, or warehouse dispatching capacities are ignored. Integrated models provide a more complete view, although they also require more data and stronger computational support.

Table 2. Comparison of major research streams

Stream	Core decisions	Main constraints	Common methods	Main limitations
SJRP	Replenishment triggers, safety stock, joint cycles.	Demand distributions, shortage costs, service levels.	MDP, heuristics, differential evolution.	Sensitive to distribution assumptions and parameter estimation.
DJRP	Multi-period order quantities, cycle variables, inventory targets.	Planning horizon, storage capacity, time-varying costs.	Dynamic programming, MILP, rolling horizon optimization.	Difficult to solve large-scale instances.
Resource-constrained JRP	Cycles, order quantities, vehicle or capacity allocation.	Budget, storage, transportation, minimum order quantity.	Genetic algorithms, differential evolution, approximation methods.	Feasible region is complex and parameters interact strongly.
Special JRP	Discount choice, credit period, substitution or defective-item handling.	Price mechanisms, quality, information errors.	Hybrid metaheuristics, sensitivity analysis.	Models are diverse and general conclusions are limited.
JRD/IRP	Replenishment frequency, delivery batches, vehicle routes.	Warehouses, vehicles, routes, service levels.	VNS, differential evolution, PPO, GNN.	Interpretability and real-time stability remain challenging.

5. MANAGERIAL IMPLICATIONS

First, the applicability of joint replenishment depends on the relative importance of shared fixed costs and holding costs. When major ordering or shipment costs are high, coordinated replenishment can create substantial economies of scale. When item-level holding costs or demand patterns differ sharply, excessive synchronization can increase inventory pressure. Managers should therefore avoid treating JRP as a universal rule for ordering all items together.

Second, the implementation quality of JRP depends on reliable operational data. Demand rates, holding costs, ordering costs, shipment costs, lead times, storage capacities, and vehicle resources are often estimated rather than known. If these estimates are inaccurate, a mathematically optimal policy may perform poorly in actual operations. Sensitivity analysis and periodic parameter updating should be incorporated into replenishment planning.

Third, algorithm selection should match the managerial setting. Small-scale and relatively stable systems may be handled by classical heuristics or mathematical programming. Large-scale systems with multiple operational constraints may require metaheuristics or hybrid algorithms. Highly dynamic environments, such as VMI networks with frequent state changes, may benefit from learning-based methods, but these methods must be accompanied by feasibility checks and interpretable decision rules.

Fourth, integrated replenishment-distribution models are particularly useful when transportation resources are expensive or service time windows are strict. In such cases, separating inventory decisions from delivery planning can lead to locally optimal but systemically inefficient decisions. JRD and IRP models help managers evaluate whether inventory savings are offset by transportation costs or operational infeasibility.

6. RESEARCH GAPS AND FUTURE DIRECTIONS

First, JRP is moving from static inventory modeling toward dynamic S&OP settings. Traditional models usually treat demand, unit costs, and transportation capacity as known constants. In real planning processes, these variables are updated frequently. Recent approximation-oriented work on resource-constrained JRP also suggests that theoretical performance guarantees remain important when operational constraints become complex [36]. Future research may further connect demand forecasting, rolling planning, replenishment optimization, and execution feedback.

Second, integrated replenishment-distribution decisions are expanding from single-warehouse systems to multi-warehouse, multi-node, and heterogeneous-product networks. Robust and distributionally robust IRP studies show how uncertain demand and risk preferences can be incorporated into inventory-routing decisions [37, 38]. Reinforcement learning has also been introduced into stochastic and dynamic IRP with fine time granularity [39]. These developments indicate that JRP-related research is gradually moving toward real-time and network-based decision environments.

Third, algorithmic research is developing along two parallel lines: learning-based optimization and explainable operational rules. Deep reinforcement learning has been increasingly discussed in inventory control and supply chain management [40, 41]. However, learning-based approaches do not automatically replace operations research models. Their training stability, constraint feasibility, sample efficiency, and generalization ability must be carefully evaluated before they can be used in operational decision support.

Table 3. Research trends, bases, and challenges

Trend	Research basis	Main challenge
Dynamic S&OP integration	Dynamic-demand JRP with time-varying costs and transportation resources links replenishment to integrated planning [17].	Forecast errors, rolling planning, and execution feedback are difficult to coordinate.
Network coordination	JRD and IRP jointly optimize replenishment, warehousing, vehicles, and routes [29, 34].	The number of variables grows quickly, and model interpretation becomes harder.
Learning-based optimization	PPO, GNN, and reinforcement learning have been applied to JRP or IRP-VMI settings [27, 35, 39].	Training stability, feasibility constraints, and generalization remain uncertain.
Multi-objective evaluation	Service, carbon emissions, risk, and robustness enter inventory-distribution models [38, 42-44].	Objective weights and management preferences are difficult to determine objectively.

Fourth, performance evaluation is shifting from cost minimization toward multi-objective decision-making. Service level, carbon emissions, risk preference, response time, and robustness are becoming more visible in replenishment and distribution models. Studies on fresh food supply chains, green IRP, and sustainable replenishment-routing systems show that environmental and service criteria can materially change replenishment decisions [42-44]. A key challenge is how to set objective weights and managerial preferences in a transparent way.

7. LIMITATIONS OF THE REVIEW

This paper is a narrative review rather than a systematic review or bibliometric analysis. It focuses on representative studies that illustrate the evolution of JRP and related inventory-distribution models. As a result, it does not report a full PRISMA-style screening process, keyword frequency analysis, citation network analysis, or statistical meta-analysis.

Another limitation is that the review is mainly based on English-language literature. Chinese-language and industry-specific studies may contain additional insights, especially in e-commerce logistics, retail distribution, cold-chain replenishment, and spare-parts supply. Future work may combine systematic database screening with bibliometric visualization to provide a more quantitative view of the research landscape.

8. CONCLUSION

This paper reviewed the joint replenishment problem and its related extensions from the perspectives of modeling assumptions, operational constraints, decision boundaries, and algorithmic paradigms. The classical JRP explains the basic cost trade-off between major ordering cost savings and inventory holding cost increases. Stochastic demand, dynamic demand, and resource-constrained models relax the assumptions of demand certainty, time invariance, and unlimited resources. JRD and IRP further extend the decision boundary from replenishment cycles to delivery schedules, vehicle routing, and network coordination.

The review indicates that recent JRP research is no longer limited to finding a low-cost replenishment cycle. More attention is being paid to uncertain demand, constrained resources, integrated distribution, data-driven planning, and multiple performance criteria. For managers, the value of JRP lies in its ability to support coordinated decisions across inventory and logistics activities. For researchers, the central challenge is to design models and algorithms that are not only computationally effective but also data-compatible, explainable, and implementable in real supply chain environments.

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